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The Valuation Portfolio

1 The purpose of this paper

The obligation of the insurer towards its beneficiaries are traditionally secured by means of actuarial reserves. Actuaries have always had the task to value these reserves. Such actuarial valuations are typically one dimensional. They produce a scalar figure (an amount e.g. in Euros) indicating how much money is needed to fulfill the obligations of the insurance contracts defining premiums to be received and benefits to be paid.

Traditionally one compares these actuarial reserves with those assets which are available to cover the insurance obligations. If the – one dimensional – value of these assets exceeds the scalar value of the actuarial reserves, the company has a so-called surplus.

The main point of this paper is the following:

i) Having a nonnegative surplus is a **necessary** but not a **sufficient** condition for an insurance company to be solvent.

ii) In order to judge the solvability of an insurance company one needs to understand both assets and liabilities as portfolios of financial instruments i.e.

A) on the asset side → portfolio of assets available for covering insurance obligations short: Investment Portfolio S

B) on the liability side → portfolio of liabilities deriving from the insurance obligations short: Valuation Portfolio (VaPo)

Solvability is then determined on the basis of the comparison of the Investment Portfolio versus the Valuation Portfolio.

Be aware: It is essential to distinguish between

a) portfolio → a list of financial instruments (titles) indicating how many of each titles you hold

b) value of portfolio → value (e.g. market value in Euros) of a)

Solvability must be judged on the basis of a).

Judging solvability on the basis of b) would mean that you compare only two figures. It seems intuitively clear that reality can not be as simple as that!

iii) On the asset side the understanding of the assets as a portfolio (list of titles) is straightforward. However, the understanding of the liabilities as a portfolio involves a new way of thinking. It is essentially the purpose of this paper to explain how insurance liabilities give rise in a very natural way to the Valuation Portfolio.

At the end of this article we give some hints how the concept of Valuation Portfolio can be used e.g. to judge solvency or to determine the embedded value.

2 The meaning of actuarial reserves

The best way to understand the meaning of reserves is the following:

Suppose you decide to transfer a block of business to another insurance carrier. The reserves that go along with this block of business should permit the new carrier to enter into the insurance obligations implied by this block. Disregarding administration costs – the new insurance carrier should just break even.

We need to be more precise in the understanding of the last sentence. For this purpose we distinguish two interpretations:

i)	break even "on average"	$\rightarrow$	this leads to best estimate reserves
ii)	break even "on average" and on financial deviations from average	$\rightarrow$	this leads to protected reserves

It will become clearer later how exactly we understand "on average". In a nutshell it means that we calculate with (conditional) expected values for the relevant insurance quantities.

3 (Pure) Valuation Portfolio in the case of best estimates

In this section we consider the insurance processes as deterministic, e.g.

- deterministic mortality table in life insurance or
- deterministic loss development pattern in non-life insurance.

This means that at all points in time we replace the stochastic insurance process by its expected counterpart.
In this world of expectations reserves have the property of a linear operator, which means that

Reserves for block A and block B	=	Sum of individual reserves for block A and B
Reserves for multiple of block A	=	Multiple of reserves of block A

Observe. This is true also for dependent blocks.
The practical consequence of the linearity property is important:
We may calculate reserves at arbitrary levels, e.g.

- individual contract (policy),
- block of business,
- entire business.

By aggregating reserves or breaking them down we can easily move from one level to the other. This fact is not only true for reserves in the classical sense. It holds as well for the respective (Pure) Valuation Portfolios that we are going to construct. For the protected reserves and the protected Valuation Portfolio the linearity breaks down (cf. Section 4).

3.1 A life insurance example

In life insurance it is convenient to construct the Valuation Portfolio for each policy and aggregating these individual VaPo's for the whole business under consideration.

Example 3.1 (Pure endowment insurance (indexed))

We consider a pure endowment insurance policy for a male person. We assume the age at the entry of the contract is $x = 50$ and the contract duration is $n = 5$ years. Moreover, we assume that

- Face value is 100 currency units.
- Benefit at survival is indexed by an known index $\mathbf{I} = (I_t)_{t=0,\ldots,5}$ of a specified portfolio of investment titles with $I_0 = 1$. The benefit at survival is given by $\max\{100, I_5 \cdot 100\}$, i.e. there is a minimal guarantee of 100 currency units in case of survival.

- Yearly premium P $(t = 0, 1, \dots, 4)$ to be paid at the beginning of each policy year after begin of the policy (given by tariff).

Hence, the pure endowment contract defines a cashflow $\mathbf{X} = (X_0, \dots, X_5)$. The financial instruments that are involved in the definition of future premiums and benefits are

$Z^{(t)}$	zero coupon bond paying one currency unit at time t $(t = 1, 2, 3, 4)$
$\mathbf{I} = (I_t)_{t=0,\dots,5}$	one unit of fund replicating the index (index fund)
$\text{Put}^{(5)}(\mathbf{I})$	put option on index fund $\mathbf{I}$ guaranteeing the value one at time $t = 5$

Thus, we have a total of 6 financial instruments (units)

$$(U_1, \dots, U_6) = \left(Z^{(1)}, \dots, Z^{(4)}, \mathbf{I}, \text{Put}^{(5)}(\mathbf{I})\right) \tag{3.1}$$

and we make a **multidimensional valuation** using these financial instruments as basis of the VaPo.

Using a deterministic life table we want to construct the VaPo at time $t = 0$ (after first premium is paid) for l_{50} persons. This means that we have to determine the number of each unit. For one person we only need to divide the number of units by l_{50}.

If we give a positive sign to payments from insurer to the insured we obtain the following *valuation scheme* at time $t = 0$ for l_{50} persons (after payment of the first premium):

time	cashflow	basis elements U_m	number of units (of basis elements)
$t = 1$	X_1	$Z^{(1)}$	$-P \cdot l_{51}$
$t = 2$	X_2	$Z^{(2)}$	$-P \cdot l_{52}$
$t = 3$	X_3	$Z^{(3)}$	$-P \cdot l_{53}$
$t = 4$	X_4	$Z^{(4)}$	$-P \cdot l_{54}$
$t = 5$	X_5	$\mathbf{I}$	$100 \cdot l_{55}$
	X_5	$\text{Put}^{(5)}(\mathbf{I})$	$100 \cdot l_{55}$

Table 1: Valuation scheme for the pure VaPo at time $t = 0$ (life insurance)

Observe. To obtain the valuation scheme at time $t = 1$ for l_{51} persons we only need to cancel the first line in the valuation scheme at time $t = 0$.

Conclusion. The valuation portfolio at time $t = 1$ is obtained exactly by adding the premiums receivable from the policy at that time. This observation is generally true. This means that if you add the cashflow from the policy the valuation portfolio is **self-financing**.

Explanations.

- The last two columns in Table 1 represent the pure VaPo at time $t = 0$. The holding of this VaPo enables the insurer to fulfill the future expected liabilities viewed from time $t = 0$. In this example the pure VaPo is a six-dimensional portfolio that represents the future expected liabilities.

- The self-financing property just described is of course only true in the deterministic model (i.e. deterministic life table).

Intermission: What is risk?
In order to measure risk, you always need a theoretical model which serves as reference. In the example above we have seen that

- if the deterministic model holds and

- if you invest in the VaPo

there is **no** risk during the lifetime of the policy (but there may have been an initial loss or gain at the beginning).
Therefore we can clearly separate the risk of this policy into

technical risk	:	deviation of mortality from the deterministic model
financial risk	:	deviation of actual investment of assets from the VaPo

3.2 A non-life insurance example

We look at the pooled data of the claims experience in a branch of general insurance. They are typically summarized in a claims development triangle (sometimes also claims trapezoids) as shown in Table 2.
In the following $X_{i,j}$ denotes the incremental payments for accident year $i \in \{1, \ldots, I\}$ in development year $j \in \{0, \ldots, J\}$. We assume that there are no further payments after J development years.

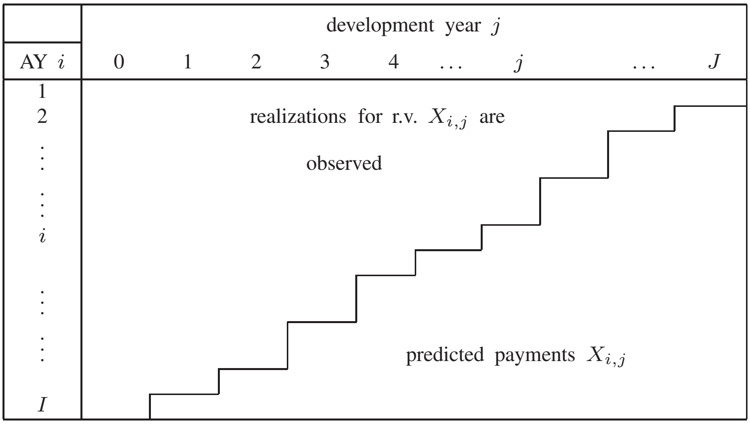

Table 2: Claims development triangle (i.e. $I = J - 1$)

The payments within a fixed accounting year are given by

$$X_k = \sum_{i+j=k} X_{i,j} = \sum_{i=1\vee(k-J)}^{k\wedge I} X_{i,k-i}\,, \tag{3.2}$$

which are the diagonals of our loss development rectangle.

Assume that we are at time $t = I$. This means that we have made the observations

$$\mathcal{T}_I = \{X_{i,j};\ i + j \leq I\}\ . \tag{3.3}$$

Claims reserving means that we need to put enough money aside (into the reserve) such that we are able to meet the future cashflows

$$X_{I+1}, X_{I+2}, X_{I+3}, \dots\,, \tag{3.4}$$

generated by the accident years $1 \leq i \leq I$. These cashflows X_{I+k} ($k \geq 1$) correspond to the sums in the unknown diagonals in the loss development rectangle, i.e.,

$$X_{I+k} = \sum_{i+j=I+k} X_{i,j} = \sum_{i=1\vee(I+k-J)}^{I} X_{i,I+k-i}\,. \tag{3.5}$$

Aim. Our aim at time $t = I$ is to construct a portfolio of financial instruments such that if we hold this portfolio we can meet all obligations. In the case of best estimates this means that we should be able to pay the expected values of these obligations.

Hence, we construct a portfolio that covers the future (conditionally) expected liabilities at time $t = I$

$$\lambda_{I+k}^{(I)} = E\left[X_{I+k} \middle| \mathcal{T}_I\right] \quad \text{for } k \geq 1\,. \tag{3.6}$$

This gives the following valuation scheme in Table 3.

time	cashflow	basis elements U_m	number of units (of basis elements)
$t = I + 1$	X_{I+1}	$Z^{(I+1)}$	$\lambda_{I+1}^{(I)}$
$t = I + 2$	X_{I+2}	$Z^{(I+2)}$	$\lambda_{I+2}^{(I)}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
$t = I + k$	X_{I+k}	$Z^{(I+k)}$	$\lambda_{I+k}^{(I)}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$

Table 3: Valuation scheme for the pure VaPo at time $t = I$ (non-life insurance)

Explanations.

- $Z^{(j)}$ stands for the unit zero coupon bond paying one currency unit at time $t = j$. These zero coupon bonds constitute the basis of the VaPo.

- The last two columns in Table 3 represent the pure VaPo at time $t = I$. Observe that the holding of this VaPo enables to fulfill the future (conditional) expected liabilities $\lambda_{I+k}^{(I)}$ viewed from time $t = I$. Henceforth, holding the pure VaPo allows for meeting the expected liabilities.

- The (pure) VaPo is a multidimensional portfolio that represents the future (conditionally) expected liabilities. Formally, this (pure) VaPo at time $t = I$ can be written as

$$\text{VaPo}(I) = \sum_{k \geq 1} \lambda_{I+k}^{(I)} \cdot Z^{(I+k)}. \tag{3.7}$$

Remark. Contrary to the VaPo in life insurance the choice of the basis elements in the non-life VaPo is less obvious. We have chosen zero coupon bonds as basis elements, since this choice is quite natural for non-deflated claims development triangles. Otherwise we could also consider zero coupon bonds indexed with inflation or even claims inflation.

Final Remark. Whereas in life insurance it seemed natural to build the VaPo for each contract and summing up for pooled data, we, typically, in non-life insurance construct the VaPo at the pooled data level directly.

3.3 The general case of the VaPo in the case of best estimates

The two special cases treated in Subsections 3.1 and 3.2 can be summarized as follows:

At time $t = I$ we reserve for the future cashflows

$$X_{I+1}, \ldots, X_{I+2}, \ldots, X_{I+k}, \ldots \tag{3.8}$$

Their (conditional) expected values are not expressed in money but in units U_m; $m \in \mathcal{M}$. These units represent properly chosen financial investments and constitute the basis of the VaPo. At time $t = I + k$ we have, hence, the following expected payments in instrument U_m

$$\lambda^{(I)}_{m,I+k} \cdot U_m \,. \tag{3.9}$$

The aggregation over all future time points $t = I + k$ ($k > 0$) and all financial instruments U_m yields the (Pure) Valuation VaPo at time $t = I$:

$$\boxed{\mathrm{VaPo}(I) = \sum_{k>0} \sum_{m\in\mathcal{M}} \lambda^{(I)}_{m,I+k} \cdot U_m} \tag{3.10}$$

Observe. In Subsection 3.2 we have chosen only **one** financial instrument for each time point $t = I + k$.

The following properties of VaPo(I) are essential:

1) If you add the cashflow resulting from the insurance contract the payments $\lambda^{(I)}_{m;I+k}$ in units U_m are guaranteed in a self-financing fashion from VaPo(I).

2) This self-financing property is only true in the (conditional) mean. In reality we would need a random number of units

$$\Lambda_{m,I+k} \text{ units } U_m \tag{3.11}$$

at each time point $t = I + k$ with $k > 0$.

Following the "Intermission: What is risk ?" in Subsection 3.1 the difference between real and (conditional) expected payments in units U_m

$$\Lambda_{m,I+k} - \lambda^{(I)}_{m,I+k} = \Delta^{(I)}_{m,I+k} \tag{3.12}$$

is called **technical risk**.

The following section studies the question how to protect against the deviations $\Delta^{(I)}_{m,I+k}$ at each time point $t = I + k$ with $k > 0$.

4 VaPo protected against technical risk

In the sequel it is essential to think of the VaPo as the one integrated for the entire block of business (see our remark in Section 3, p. 71 on the non-additivity of the protected VaPo).
Following the exposition in Subsection 3.3 we have for the (pure) VaPo

$$\text{VaPo}(I) = \sum_{k>0} \sum_{m\in\mathcal{M}} \lambda^{(I)}_{m,I+k} \cdot U_m \,. \tag{4.1}$$

In order to finance the deviations in the different time points $t = I + k$ $(k > 0)$

$$\Delta^{(I)}_{m,I+k} \tag{4.2}$$

we increase the number of units U_m from $\lambda^{(I)}_{m,I+k}$ to $\lambda^{*(I)}_{m,I+k}$ and obtain the so-called **VaPo protected against technical risk**

$$\boxed{\text{VaPo}^{prot}(I) = \sum_{k>0} \sum_{m\in\mathcal{M}} \lambda^{*(I)}_{m,I+k} \cdot U_m} \tag{4.3}$$

You may think of this increase from $\lambda^{(I)}_{m,I+k}$ to $\lambda^{*(I)}_{m,I+k}$ in different ways

a) in a more theoretical framework

$\lambda_{m,I+k}^{(I)}$ is a (conditional) expected value and
$\lambda_{m,I+k}^{*(I)}$ is a (conditional) certainty equivalent,

b) more practically oriented you may consider

$$\lambda_{m,I+k}^{*(I)} - \lambda_{m,I+k}^{(I)} \tag{4.4}$$

as reinsurance premium, payable in either unit U_m for a **run-off reinsurance cover** or

c) you may consider

$$\lambda_{m,I+k}^{*(I)} - \lambda_{m,I+k}^{(I)} \tag{4.5}$$

in either unit U_m as **cost of capital** for absorbing the differences $\Delta_{m,I+k}^{(I)}$ into your risk capital.

Remark. In general $\lambda_{m,I+k}^{*(I)} - \lambda_{m,I+k}^{(I)}$ is determined by the choice of a utility function (case a)) or on the basis of a risk measure (cases b) and c)). Therefore the protection against technical risk is to be understood as relative to the chosen utility function or risk measure.

Once we have made the step from VaPo(I) to VaPo$^{prot}(I)$ we are protected against technical risk. The only risk that remains is the **financial risk** deriving from the fact that your

- actual investment portfolio *S* and
- the valuation portfolio VaPo^{prot}

may differ. This risk is addressed in the last section under 6.2.

5 A numerical example (non-life insurance)

For a numerical illustration we use the example treated in [2]. For details we refer to that paper.
We start with the observed triangle of incremental loss payments $X_{i,j}$, where $X_{i,j}$ denotes incremental loss payments for accident year i and development year j (cf. Table 4).
The corresponding accounting year payments X_k are given in Table 5.

i/j	0	1	2	3	4	5	6	7	8	9
1	357.848	766.940	610.542	482.940	527.326	574.398	146.342	139.950	227.229	67.948
2	352.118	884.021	933.894	1'183.289	445.745	320.996	527.804	266.172	425.046	
3	290.507	1'001.799	926.219	1'016.654	750.816	146.923	495.992	280.405		
4	310.608	1'108.250	776.189	1'562.400	272.482	352.053	206.286			
5	443.160	693.190	991.983	769.488	504.851	470.639				
6	396.132	937.085	847.498	805.037	705.960					
7	440.832	847.631	1'131.398	1'063.269						
8	359.480	1'061.648	1'443.370							
9	376.686	986.608								
10	344.014									

Table 4: Observed triangle of incremental loss payments $X_{i,j}$

k	1	2	3	4	5	6	7	8	9	10
X_k	357.8	1'119.1	1'785.1	2'729.2	4'188.2	3'902.3	5'150.5	3'911.3	5'221.1	5'993.5

Table 5: Accounting year payments X_k

Using the well-known chain-ladder technique we get the predicted expected figures $\widehat{E}[X_{i,j}|\mathcal{T}_I]$ in the lower triangle (cf. Table 6).

This leads to the predicted expected payments

$$\widehat{\lambda_{I+k}^{(I)}} = \sum_{\substack{i+j=I \\ 0 \leq j+k \leq J}} \widehat{E}[X_{i,j+k}|\mathcal{T}_I]$$

for the different accounting years $t = I + k$ viewed from time $t = I = 10$ (cf. Table 7).

Thus we obtain immediately the (pure) Valuation Portfolio at time point $t = I$ (cf. Table 8).

For the VaPo protected against technical risk the number of units $\widehat{\lambda_{I+k}^{(I)}}$ is increased by adding the cost of risk capital (cf. possibility c) in Section 4). As explained in [2] the

i/j	0	1	2	3	4	5	6	7	8	9
1										
2										94.634
3									375.833	93.678
4								247.190	370.179	92.268
5							334.148	226.674	339.456	84.611
6						383.287	351.548	283.477	357.132	89.016
7					605.548	424.501	389.349	264.121	395.534	98.588
8				1'310.258	725.788	508.792	466.660	316.566	474.073	118.164
9			1'018.834	1'089.616	603.569	423.113	388.076	263.257	394.241	98.266
10		856.804	897.410	959.756	531.636	372.687	341.826	231.882	347.255	86.555

Table 6: Predicted expected figures $\widehat{E}[X_{i,j}|\mathcal{T}_I]$

$I+k$	11	12	13	14	15	16	17	18	19
$\widehat{\lambda^{(I)}_{I+k}}$	5'226.5	4'179.4	3'131.7	2'127.3	1'561.9	1'177.7	744.3	445.5	86.6

Table 7: Predicted expected payments $\widehat{\lambda^{(I)}_{I+k}}$

time	basis elements U_m	number of units $\widehat{\lambda^{(I)}_{I+k}}$
$t = 11$	$Z^{(11)}$	5'226.5
$t = 12$	$Z^{(12)}$	4'179.4
$t = 13$	$Z^{(13)}$	3'131.7
$t = 14$	$Z^{(14)}$	2'127.3
$t = 15$	$Z^{(15)}$	1'561.9
$t = 16$	$Z^{(16)}$	1'177.7
$t = 17$	$Z^{(17)}$	744.3
$t = 18$	$Z^{(18)}$	445.5
$t = 19$	$Z^{(19)}$	86.6

Table 8: (Pure) Valuation Portfolio at time $t = I = 10$

- risk capital $\rho^{(I)}(X_{I+k})$ for time $t = I + k$ is chosen proportional to $\sqrt{\widehat{\text{msep}}\left(\widehat{\lambda_{I+k}^{(I)}}\right)}$, where $\widehat{\text{msep}}\left(\widehat{\lambda_{I+k}^{(I)}}\right)$ is an estimate of the Mean Square Error of Prediction for $\widehat{\lambda_{I+k}^{(I)}}$ and
- r stands for the cost rate charged (8%) on the risk capital.

Thus we have

$$\widehat{\lambda_{I+k}^{*(I)}} = \widehat{\lambda_{I+k}^{(I)}} + r \cdot \rho^{(I)}(X_{I+k})$$

for all $k > 0$.
For the risk measure we choose

$$\rho^{(I)}(X_{I+k}) = \beta \cdot \sqrt{\widehat{\text{msep}}\left(\widehat{\lambda_{I+k}^{(I)}}\right)}$$

with $\beta = 2.326 = \Phi^{-1}(0.99)$ (Φ denotes the standard Gaussian distribution). This gives the following numerical values:

$I + k$	11	12	13	14	15	16	17	18	19
$\widehat{\lambda_{I+k}^{(I)}}$	5’226.5	4’179.4	3’131.7	2’127.3	1’561.9	1’177.7	744.3	445.5	86.6
$\rho^{(I)}(X_{I+k})$	1’548.3	1’545.3	1’464.2	1’148.0	913.9	666.6	406.1	358.3	172.9
$\widehat{\lambda_{I+k}^{*(I)}}$	5’350.4	4’303.0	3’248.8	2’219.1	1’635.0	1’231.1	776.8	474.2	100.4

Table 9: Number of units $\widehat{\lambda_{I+k}^{(I)}}$ and $\widehat{\lambda_{I+k}^{*(I)}}$ and risk capital $\rho^{(I)}(X_{I+k})$

This leads to the Valuation Portfolio protected against technical risk at time $t = I = 10$ (cf. Table 10).

6 The use of the VaPo

In our paper we have concentrated on explaining how to express the obligations of the insurer as a portfolio. More specifically, we have shown how to construct

VaPo(I)	(Pure) Valuation Portfolio at time $t = I$ and
VaPo$^{prot}(I)$	Valuation Portfolio protected against technical risk at time $t = I$.

In this final section we list the most important applications where the VaPo plays a central role.

time	basis elements U_m	number of units $\widehat{\lambda_{I+k}^{*(I)}}$
$t = 11$	$Z^{(11)}$	5’350.4
$t = 12$	$Z^{(12)}$	4’303.0
$t = 13$	$Z^{(13)}$	3’248.8
$t = 14$	$Z^{(14)}$	2’219.1
$t = 15$	$Z^{(15)}$	1’635.0
$t = 16$	$Z^{(16)}$	1’231.1
$t = 17$	$Z^{(17)}$	776.8
$t = 18$	$Z^{(18)}$	474.2
$t = 19$	$Z^{(19)}$	100.4

Table 10: Valuation Portfolio protected against technical risk at time $t = I = 10$

6.1 Embedded value in life insurance

This derives from the fact that different accounting principles give different values to the VaPo or the VaPo^{prot}. For more details see [1], [3], [4] and [5].

6.2 Guaranteeing Solvency

The insurance company is solvent, if at all time points $t = I + k$ $(k \geq 0)$ it could switch from the actual investment portfolio *S* to the VaPo^{prot}. This switching can be secured by

a) **Margrabe Options.** For more details see [2], [3], [6] and [7].

b) **Additional risk capital.** Fore more details see [3], [4], [5] and [8].

Of course the main point is more general: Expressing liabilities as a portfolio of financial instruments renders assets and liabilities comparable. The comparison of liabilities in the usual form with assets typically measured by market values and liabilities measured by traditional actuarial techniques is like comparing apples with pears (i.e. makes no sense).

Bibliography

[1] Baumgartner, G., Bühlmann, H., Koller, M. (2004). Multidimensional valuation of life insurance policies and fair value. SAV Bulletin, pp. 27–64.

[2] Buchwalder, M., Bühlmann, H., Merz, M., Wüthrich, M. V. (2006). Valuation portfolio in non-life insurance. Accepted for publication in Scandinavian Actuarial Journal 2007. Preprint, ETH Zürich.
http://www.math.ethz.ch/∼wueth/Papers/VaPo1.pdf.

[3] Bühlmann, H. (2004). Multidimensional valuation. Finance 25, pp. 15–29.

[4] De Felice, M., Moriconi, F. (2004). Market consistent valuation in life insurance. Measuring fair value and embedded options. Giornale dell'istituto italiano degli attuari, LXVII, Roma, pp. 95–117.

[5] De Felice, M., Moriconi, F. (2005). Market based tools for managing the life insurance company. Astin Bulletin 35/1, pp. 79–111.

[6] Gerber, H. U., Shiu, E. S. W. (1994). Option pricing by Esscher transforms. Transaction of Society of Actuaries 46, pp. 99–140.

[7] Margrabe, W. (1978). The value of an option to exchange one asset for another. Journal of Finance 33/1, pp. 177–186.

[8] Wüthrich, M. V. (2006). Vom Swiss Solvency Test zum Risikomanagement-Tool für Nicht-Lebenversicherungen. In: Swiss Solvency Test, Eine Herausforderung auf vielen Ebenen, Ernst & Young, Zürich, pp. 30–34.

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Abstract

The purpose of this paper is 1) to give a non-technical introduction into the understanding of the obligations of an insurer as a portfolio and 2) to show how we can construct this Valuation Portfolio (VaPo) for life insurance as well as for non-life insurance in a natural way. The VaPo represents the obligations of the insurer for the whole period of his insurance contracts. However, unlike to the traditional actuarial valuation these obligations are not simply measured by a scalar figure but they are expressed as a portfolio of financial instruments. Hence, the actuarial reserves become a multidimensional portfolio which we can compare with the investment portfolio. Solvability is then determined on the basis of the comparison of two portfolios rather than only two figures like in the classical actuarial valuation. Moreover, in this setup financial and technical risk are clearly separated.

Zusammenfassung

Ziel dieses Artikels ist es, erstens: eine nicht-technische Einführung zum Verstehen der Verpflichtungen eines Versicherers als ein Portfolio zu geben, und zweitens: zu zeigen, wie ein solches Bewertungsportfolio (VaPo) sowohl für einen Lebensversicherer als auch für einen Nichtleben-Versicherer auf natürliche Weise konstruiert werden kann. Das VaPo stellt die Verpflichtungen des Versicherers über die ganze Dauer seiner Versicherungsverträge dar. Anders als bei der traditionellen aktuariellen Bewertung werden diese Verpflichtungen nicht einfach durch Zahlen dargestellt sondern als ein Portfolio von Finanzinstrumenten. Aktuarielle Rückstellungen werden so zu einem multidimensionalen Portfolio, welches wir mit einem Investmentportfolio vergleichen können. Solvabilität wird dann auf der Basis des Vergleichs zweier Portfolios bestimmt, und nicht mehr durch den Vergleich zweier Zahlen, wie in der klassischen aktuariellen Bewertung. Darüber hinaus sind in dieser Darstellung technische und finanzielle Risiken klar getrennt.

Résumé

L'objectif de cet article est premièrement de donner une introduction non-technique à la compréhension des engagements des assureurs en temps que portefeuille et deuxièmement de montrer comment construire de manière naturelle ledit portefeuille d'évaluation (Vapo) d'un assureur vie et d'un assureur dommage. Le Vapo représente les engagements d'un assureur sur toute la durée de ses contrats d'assurance. Contrairement à l'évaluation actuarielle traditionnelle les engagements ne sont pas mesurés par des nombres mais par un portefeuille d'instruments financiers. Les provisions actuarielles deviennent ainsi un portefeuille à plusieurs variables que nous pouvons comparer aux portefeuilles des investissements. La solvabilité est alors déterminée par la comparaison de deux portefeuilles plutôt que par la comparaison actuarielle classique de deux chiffres. Cette représentation permet de surcroît une séparation claire des risques financier et technique.