Replicating Portfolios
Complex modelling made simple

by Jolanta Tubis

10 September 2010
Agenda

- Smart modelling
- The replicating portfolio
  - Approach
  - Case studies
Why smart modelling?

- Life insurance is about hedging exposures
- For savings products life insurance is about **individualised** guarantees
  - Each policyholder has potentially different strike prices (=guaranteed benefits), terms and benefits types
  - The resulting overall exposure for the insurance company is
    - complex
    - non-linear
    - difficult to manage
- But this gives life insurance a unique selling proposition
- In fact a life insurance portfolio is a portfolio of options
- But how does this portfolio look like?
Why smart modelling?

We have some information, but not enough and not often enough.

In order to manage the liabilities, we need:
- the whole picture
- daily

And we have issues with accuracy and run-time.

EEV gives us some insight from time to time, but with great effort. Can we use the EEV-efforts to get more? Can we reduce effort and improve accuracy?
There are many promising approaches to solve some of our technical problems

- **Weighted Monte Carlo**
  - Improves accuracy and fit to calibration
  - Still requires stochastic calculations for each piece of information

- **Change of measure – importance sampling**
  - Relevant, if not necessary, for stochastic determination of economic capital
  - Can be very successfully combined with the replication portfolio approach

- **Control variates**
  - Improves accuracy and fit to calibration
  - Still requires stochastic calculations for each piece of information

- **Moment matching**
  - Improves accuracy
  - Still requires stochastic calculations for each piece of information

- **Replicating portfolios**
  - Improves accuracy and fit to calibration
  - Can be used as control variate
  - Easy to understand and apply
  - Some relevant information can be determined without stochastic runs
  - **Enables timely and relevant management information**
Agenda

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What is a replicating portfolio?

- A replicating portfolio is a portfolio (of assets) that agrees in value with your liabilities under a range of economic conditions (=scenarios)

![Diagram showing replicating portfolio and stochastic cash-flows]

- Portfolio of non-traded options – e.g. asset share options
- Portfolio of tradable options – e.g. swaptions
- Portfolio of functions of the scenarios – e.g. annuity functions
The key problem is the determination of the „candidate assets“…

- The „candidate assets“ should be able to reflect all relevant features of the contingent cash-flows, like:
  - Dependency on core asset classes
  - Dependency on interest rates
  - Path-dependent features like e.g.
    - smoothing of returns
    - look-back-features

- Typically following candidate assets are sufficient:
  - The underlying core asset classes (in contract currency)
  - Zero bonds
  - Swaptions
  - Plain vanilla call and put options

- For a relevant range of strike prices and terms
- In some circumstances path dependent options are required
- -> Actuarial judgement is important
  - To avoid overfitting
After determining the candidate assets we can determine a portfolio as linear combination that is highly correlated with the liabilities

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Cash flow at time t</th>
<th>Asset 1</th>
<th>Asset 2</th>
<th>Asset 3</th>
<th>Asset 4</th>
<th>Asset 5</th>
<th>Asset 6</th>
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<td>L₅</td>
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</table>

Value of asset in scenario

- L₁ = w₁*A₁,₁ + w₂*A₁,₂ + w₃*A₁,₃ + ...
- L₂ = w₁*A₂,₁ + w₂*A₂,₂ + w₃*A₂,₃ + ...
- L₃ = w₁*A₃,₁ + w₂*A₃,₂ + w₃*A₃,₃ + ...
- ...

Subject to constraints...

“replicating portfolio” used as the basis of the estimation of the sensitivity
Replicating portfolios can be derived from standard liability model runs

We still need a base run
Replicating portfolios can simplify your life substantially

- The approach enables you to
  - Recalculate results for changed market parameters (asset prices, interest rates, volatility etc.)
  - Calculate sensitivities (greeks like delta, vega, rho etc.)
  - Improve accuracy and reduce the number of necessary runs
  - Project asset-dependent variables, e.g. required capital, in stochastic runs
- **…Without the need to re-project the liabilities**
  - Which is usually the onerous part of the simulation
- But the most important advantage is the fact that a replicating portfolio simplifies communication dramatically
- A replication portfolio is a description of your liabilities in terms of assets
### A typical replicating portfolio

<table>
<thead>
<tr>
<th>Candidate Asset</th>
<th>Notional in bn EUR</th>
<th>Current value</th>
<th>Value in 1 moth under stress test XYZ</th>
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<td>Swaption – EUR – 10 years term – 10 years tenor – Strike 4%</td>
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<td>...</td>
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<td>2</td>
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<td>...</td>
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<tr>
<td>Put Option on DAX – 10 years – Strike 500</td>
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<td>1</td>
<td>3</td>
</tr>
<tr>
<td>Floating strike lookback option on DAX – 20 years – strike 1234</td>
<td>3</td>
<td>2</td>
<td>2.5</td>
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<td><strong>Total</strong></td>
<td><strong>123</strong></td>
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<td><strong>45</strong></td>
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</table>
Replicating portfolios are in fact control-variates

Stochastic cashflows = replicating portfolio (closed form solution) + “residuals” (low volatility = high accuracy) Estimation error \( \approx \frac{\sigma}{\sqrt{n}} \)

A replicating portfolio of the liabilities forms an ideal control variate
Goodness of fit – typical analysis

<table>
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<tr>
<th>Description</th>
<th>Value</th>
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<tr>
<td>Exact value, closed form solution</td>
<td>124.62</td>
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<tr>
<td>Value, market-consistent scenarios</td>
<td>128.33</td>
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<tr>
<td>Value of the replicating portfolio</td>
<td>124.66</td>
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</table>
Specific issues – replicating portfolio for required economic capital

- In general the calculation of required capital requires full stochastic approach (nested stochastic simulations)
- The replicating portfolio approach allows to avoid the stochastic valuations and therefore to reduce the number of necessary calculations substantially
Specific issues – replicating portfolio for required economic capital

- The approximation must be good in the quantile considered – not only around the median
  - The optimisation approach typically enforces a good fit around the median
  - This is where most scenarios are
  - Not such a good approach for required economic capital purposes…
- Large market shocks should be replicated adequately
- It is important to ensure that the asymptotic behaviour of the replication portfolio cash-flows are reasonable
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Applications of the replicating portfolio approach

- VA portfolios- hedging
- SST target capital for a block of GMxBs – European reinsurer
- Typical German with profits business – the book value effect
Replicating portfolio approach for VA dynamic hedging

- Numerical problems
  - The Greeks calculation usually requires repeated stochastic simulation for a large number of scenarios over a huge portfolio of contracts
    - time-consuming
    - numerical errors
  - Hedge effectiveness testing usually requires nested stochastic simulations

- Our research proves
  - The Replicating portfolio approach can be successfully used to fit VA business with
    - Complex path dependent policyholder behaviour
    - Complex guarantee and asset mix structure
  - The replicating assets are plain vanilla assets that will allow for
    - Faster and more accurate valuation
    - Hedging and market risk estimation, meaning derivation of Greeks
    - Hedge effectiveness testing without time consuming nested stochastic simulations
Replicating assets

- Replicating assets
  - Plain vanilla put options
  - Basket options (including the forward starting versions) on actual underlying
    - Simulating the asset mix of the underlying asset portfolio through combinations of 70-80% equity and 30-20% bonds
  - Knock-out basket options
    - When index level exceeds a certain level the option is knocked out, if the index never exceeds the knock-out level the option is in-force

- Closed forms or numerical approximations available
Results of central projection
Scatter plot: Central scenarios

- Replicating portfolio cash flows
- MoSes cash flows

R2 Measure: 0.96
VRM: 5.16
Results of central projection
Scatter plot: Equity stress scenarios

- Replicating portfolio cash flows
- MoSes cash flows

R2 Measure: 0.96
VRM: 5.24
Results of central projection
Scatter plot: Interest stress scenarios

- Replicating portfolio cash flows
- MoSes cash flows

R² Measure: 0.97
VRM: 5.47
Applications of the replicating portfolio approach

- VA portfolios- hedging
- SST target capital for a block of GMxBs – European reinsurer
- Typical German with profits business – the book value effect
SST target capital for a block of GMxBs – European reinsurer

- Case study from 2005 (!)
- Includes policyholder behaviour (lapsation)
- Liabilities not straightforward: ratchets included
  - Thus the replication portfolio included floating strike discrete lookback options
    - Good approximation formula available
- Used for SST purposes
  - Valid approach as asymptotic behaviour is clear!
Plain vanilla contracts, no policyholder behaviour
Plain vanilla contracts, with policyholder behaviour
The replication portfolio included floating strike discrete lookback options

<table>
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<tr>
<th>Assets</th>
<th>Strike</th>
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<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
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<th>7</th>
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</tbody>
</table>
Goodness of fit

Scatter of actual versus replicating portfolio cash flows,
2nd 200 scenarios, calibrated on 100 scenarios

- Replicating portfolio cash flow
- Actual cash flow
Applications of the replicating portfolio approach

- VA portfolios- hedging
- SST target capital for a block of GMxBs – European reinsurer
- Typical German with profits business – the book value effect
Fallstudie: RP für das deutsche gewinnberechtigte Geschäft

- Gesellschaft ABC schreibt hauptsächlich das typische deutsche gewinnberechtigte Geschäft: Kapital und Renten
- Für die Fallstudie wurden die Zahlen „anonymisiert“
- Approximation des VIFs unter MCEV

$R^2$ von 95% zeigt eine gute Anpassung des RP zu dem Dividenden-Cash Flow

$y = x$

$R^2 = 0.9525$
Repricing: Um die Qualität zu verifizieren prüfen wir, ob wir mittels RP die ökonomischen Sensitivitäten replizieren können

Die Sensitivität Zins+1% ist beim RP unterschätzt

Das geschätzte RP repliziert sehr gut die Sensitivitäten: Aktien -10% und Zinsen -1%
Geschätztes RP erlaubt mühelos das ganze Spektrum der Sensitivitäten zu berechnen
RP spiegelt das Risiko des deutschen gewinnberechtigten Geschäftes wider

- MCEV kann man mit „long“-Positionen in Aktien/Immobilien/Bonds und „short“-Positionen in Derivate replizieren
- Relativ niedriger Anteil der Swaptions liegt an „Moneyness“ der Swaptions (deep out-of-the-money)
Contact

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