Replicating Portfolios
Complex modelling made simple

SAV Versammlung

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Agenda

- Smart modelling
- The replicating portfolio
  - Approach
  - Case studies

Why smart modelling?

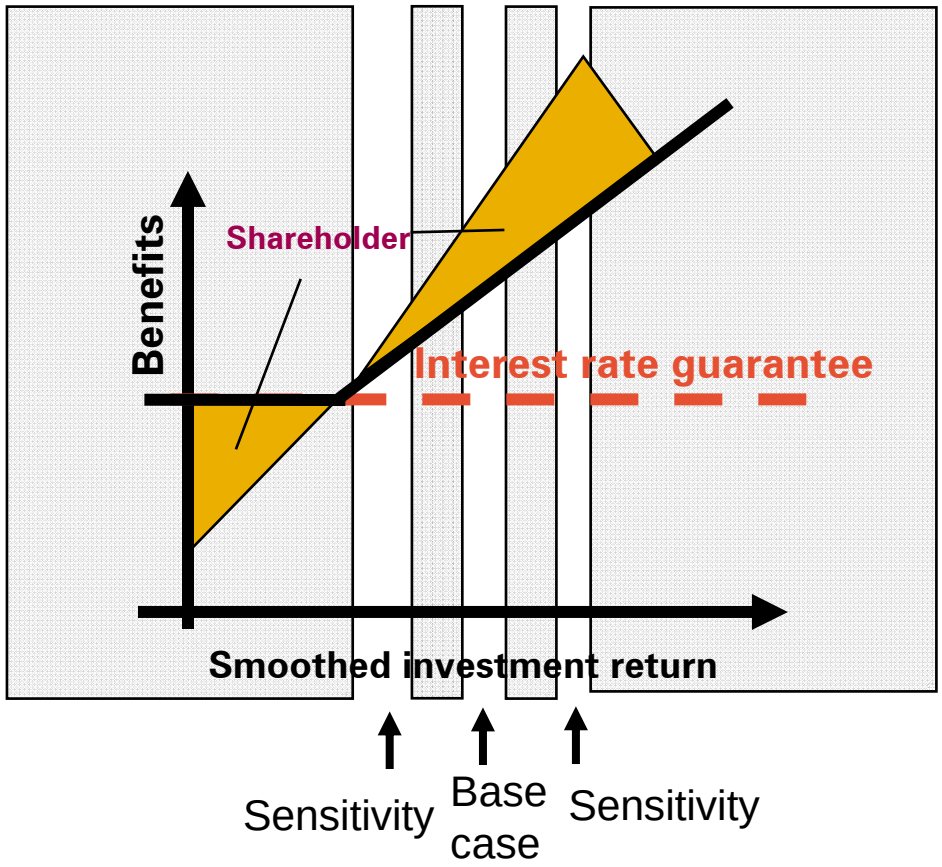
- Life insurance is about hedging exposures
- For savings products life insurance is about **individualised** guarantees
  - Each policyholder has potentially different strike prices (=guaranteed benefits), terms and benefits types
  - The resulting overall exposure for the insurance company is
    - complex
    - non-linear
    - difficult to manage
  - But this gives life insurance a unique selling proposition
  - In fact a life insurance portfolio is a portfolio of options
  - But how does this portfolio look like?

Why smart modelling?

We have some information, but not enough and not often enough

In order to manage the liabilities
we need:
- the whole picture
- daily

And we have issues with accuracy and run-time



EEV gives us some insight from time to time, but with great effort.
Can we use the EEV-efforts to get more? Can we reduce effort and improve accuracy?

There are many promising approaches to solve some of our technical problems

- **Weighted Monte Carlo**
  - Improves accuracy and fit to calibration
  - Still requires stochastic calculations for each piece of information
- **Change of measure – importance sampling**
  - Relevant, if not necessary, for stochastic determination of economic capital
  - Can be very successfully combined with the replication portfolio approach
- **Control variates**
  - Improves accuracy and fit to calibration
  - Still requires stochastic calculations for each piece of information
- **Moment matching**
  - Improves accuracy
  - Still requires stochastic calculations for each piece of information
- **Replicating portfolios**
  - Improves accuracy and fit to calibration
  - Can be used as control variate
  - Easy to understand and apply
  - Some relevant information can be determined without stochastic runs
  - **Enables timely and relevant management information**

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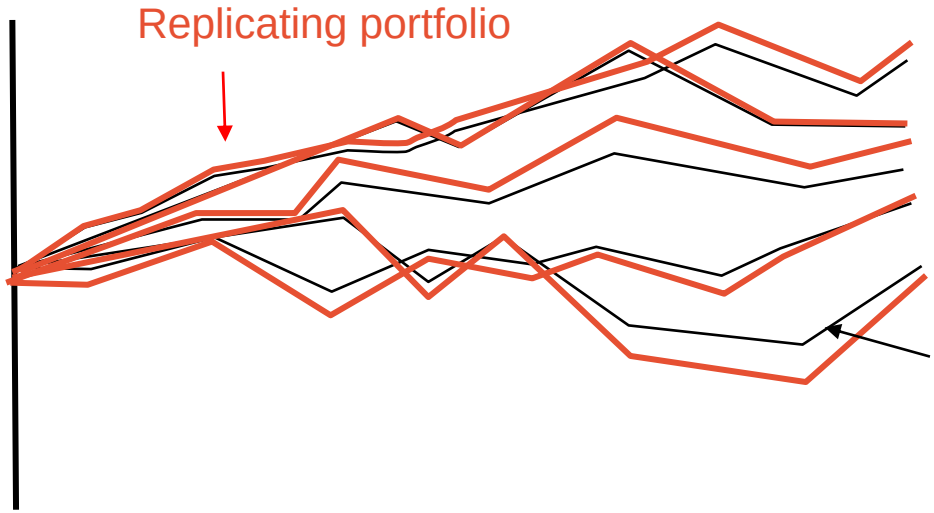
What is a replicating portfolio?

- A replicating portfolio is a portfolio (of assets) that agrees in value with your liabilities under a range of economic conditions (=scenarios)



Portfolio of non-traded options – e.g. asset share options

Portfolio of tradable options – e.g. swaptions

Portfolio of functions of the scenarios – e.g. annuity functions

The key problem is the determination of the „candidate assets“…

- The „candidate assets“ should be able to reflect all relevant features of the contingent cash-flows, like
  - Dependency on core asset classes
  - Dependency on interest rates
  - Path-dependent features like e.g.
    - smoothing of returns
    - look-back-features
- Typically following candidate assets are sufficient:
  - The underlying core asset classes (in contract currency)
  - Zero bonds
  - Swaptions
  - Plain vanilla call and put options
- For a relevant range of strike prices and terms
- In some circumstances path dependent options are required
- -> Actuarial judgement is important
  - To avoid overfitting

After determining the candidate assets we can determine a portfolio as linear combination that is highly correlated with the liabilities

		Value of asset in scenario					
Scenario	Cash flow at time t	Asset 1	Asset 2	Asset 3	Asset 4	Asset 5	Asset 6
1	L_1	$A_{1,1}$	$A_{1,2}$	$A_{1,3}$	$A_{1,4}$	$A_{1,5}$	$A_{1,6}$
2	L_2	$A_{2,1}$	$A_{2,2}$	$A_{2,3}$	$A_{2,4}$	$A_{2,5}$	A
3	L_3	$A_{3,1}$	$A_{3,2}$	$A_{3,3}$	$A_{3,4}$		
4	L_4	$A_{4,1}$	$A_{4,2}$	A			
5	L_5						

$L_1 = w_1*A_{1,1} + w_2*A_{1,2} + w_3*A_{1,3}+\ldots$

$L_2 = w_1*A_{2,1} + w_2*A_{2,2} + w_3*A_{2,3}+\ldots$

$L_3 = w_1*A_{3,1} + w_2*A_{3,2} + w_3*A_{3,3}+\ldots$

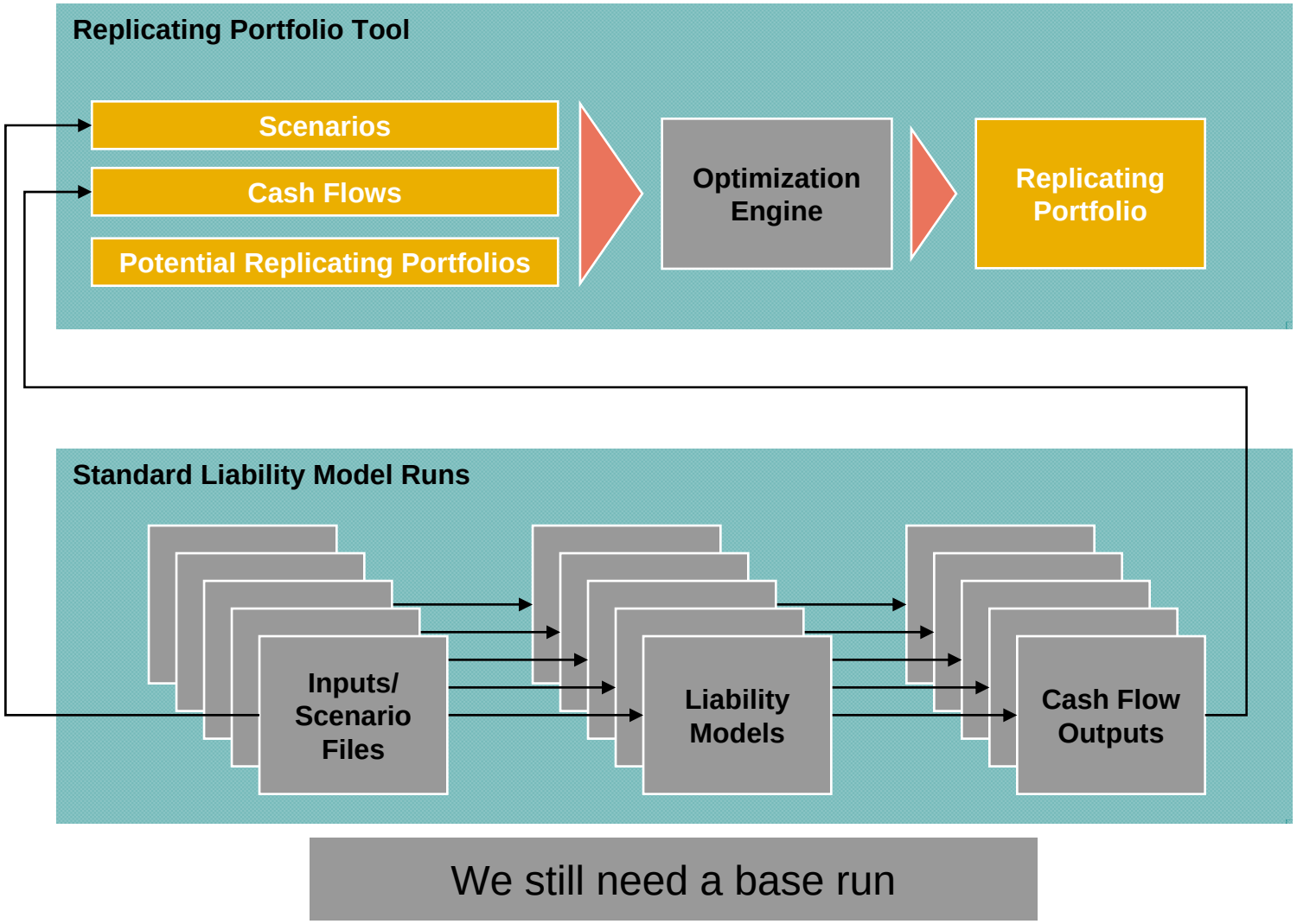
……

Subject to constraints…

"replicating portfolio"

"replicating portfolio" used as the basis of the estimation of the sensitivity

Replicating portfolios can be derived from standard liability model runs



We still need a base run

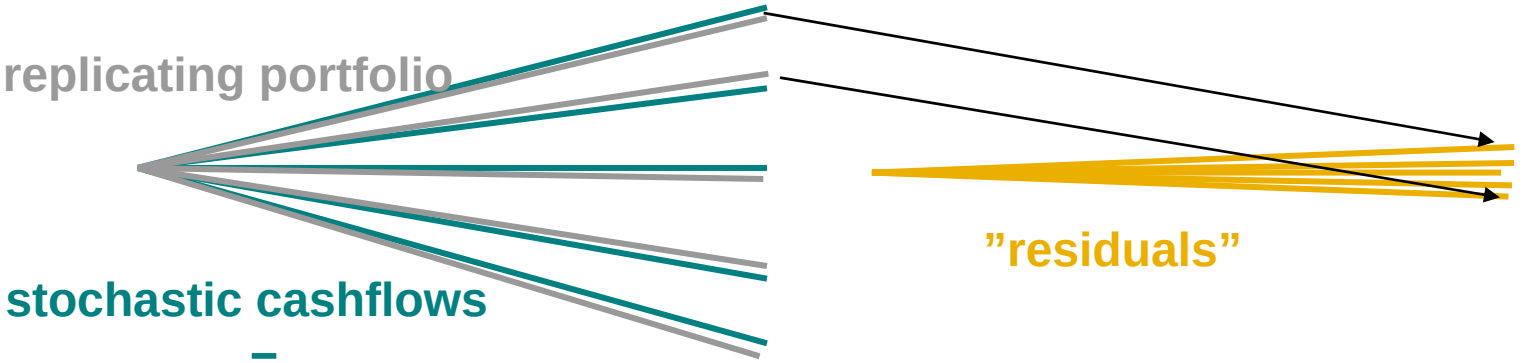
Replicating portfolios can simplify your life substantially

- The approach enables you to
  - Recalculate results for changed market parameters (asset prices, interest rates, volatility etc.)
  - Calculate sensitivities (greeks like delta, vega, rho etc.)
  - Improve accuracy and reduce the number of necessary runs
  - Project asset-dependent variables, e.g. required capital, in stochastic runs
- **…Without the need to re-project the liabilities**
  - Which is usually the onerous part of the simulation
- But the most important advantage is the fact that a replicating portfolio simplifies communication dramatically
  - A replication portfolio is a description of your liabilities in terms of assets

A typical replicating portfolio

Candidate Asset	Notional in bn EUR	Current value	Value in 1 moth under stress test XYZ
DAX	1	4	2
SMI	0.5	2	12
Zero Bond EUR – 1 year	2	2	2
…			
Zero Bond EUR – 30 years	5	3.5	3.5
Swaption – EUR – 10 years term – 10 years tenor – Strike 4%	7	0.5	0.5
…			
Swaption – EUR – 1 year term – 5 years tenor – Strike 2%	5	0.2	0.2
Put Option on DAX – 10 years – Strike 1234	10	1	2
…			
Put Option on DAX – 10 years – Strike 500	2	1	3
Floating strike lookback option on DAX – 20 years – strike 1234	3	2	2.5
Total		**123**	**45**

Replicating portfolios are in fact control-variates



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replicating portfolio (closed form solution)

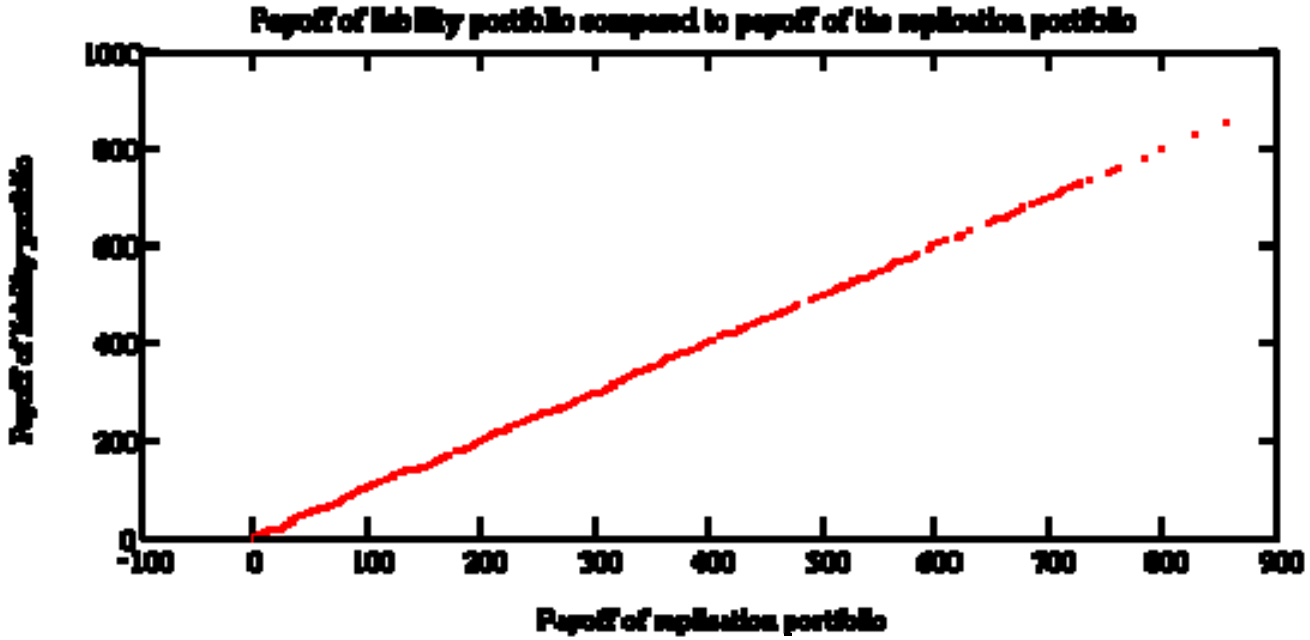
+

”residuals” (low volatility = high accuracy)

Estimation error $\approx \frac{\sigma}{\sqrt{n}}$

A replicating portfolio of the liabilities forms an ideal control variate

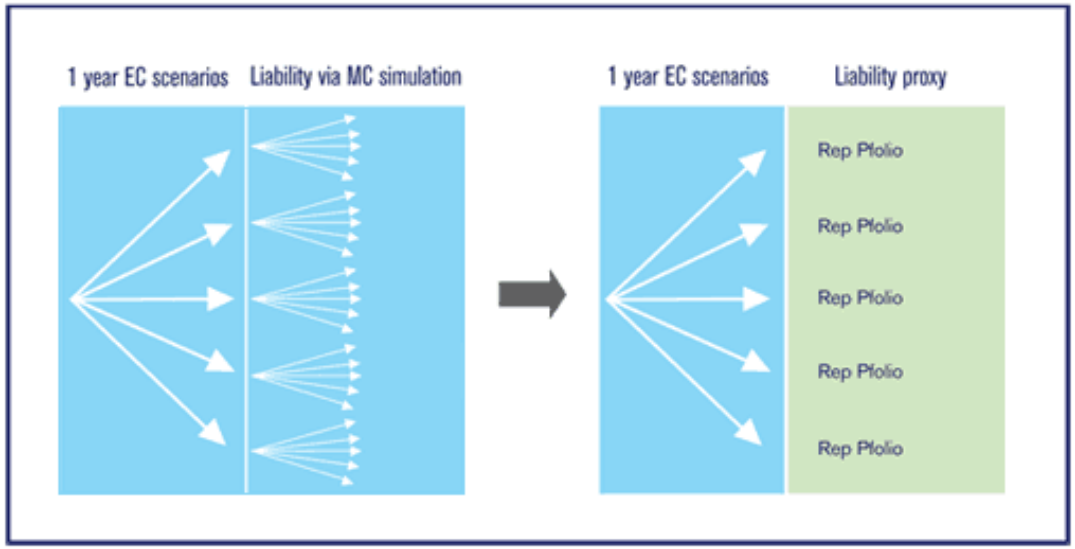
Goodness of fit – typical analysis



Exact value, closed form solution	124.62
Value, market-consistent scenarios	128.33
Value of the replicating portfolio	124.66

Specific issues – replicating portfolio for required economic capital

- In general the calculation of required capital requires full stochastic approach (nested stochastic simulations)
- The replicating portfolio approach allows to avoid the stochastic valuations and therefore to reduce the number of necessary calculations substantially



Specific issues – replicating portfolio for required economic capital

- The approximation must be good in the quantile considered – not only around the median
  - The optimisation approach typically enforces a good fit around the median
  - This is where most scenarios are
  - Not such a good approach for required economic capital purposes…
- Large market shocks should be replicated adequately
- It is important to ensure that the asymptotic behaviour of the replication portfolio cash-flows are reasonable

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Applications of the replicating portfolio approach

- VA portfolios- hedging
- SST target capital for a block of GMxBs – European reinsurer
- Typical German with profits business – the book value effect

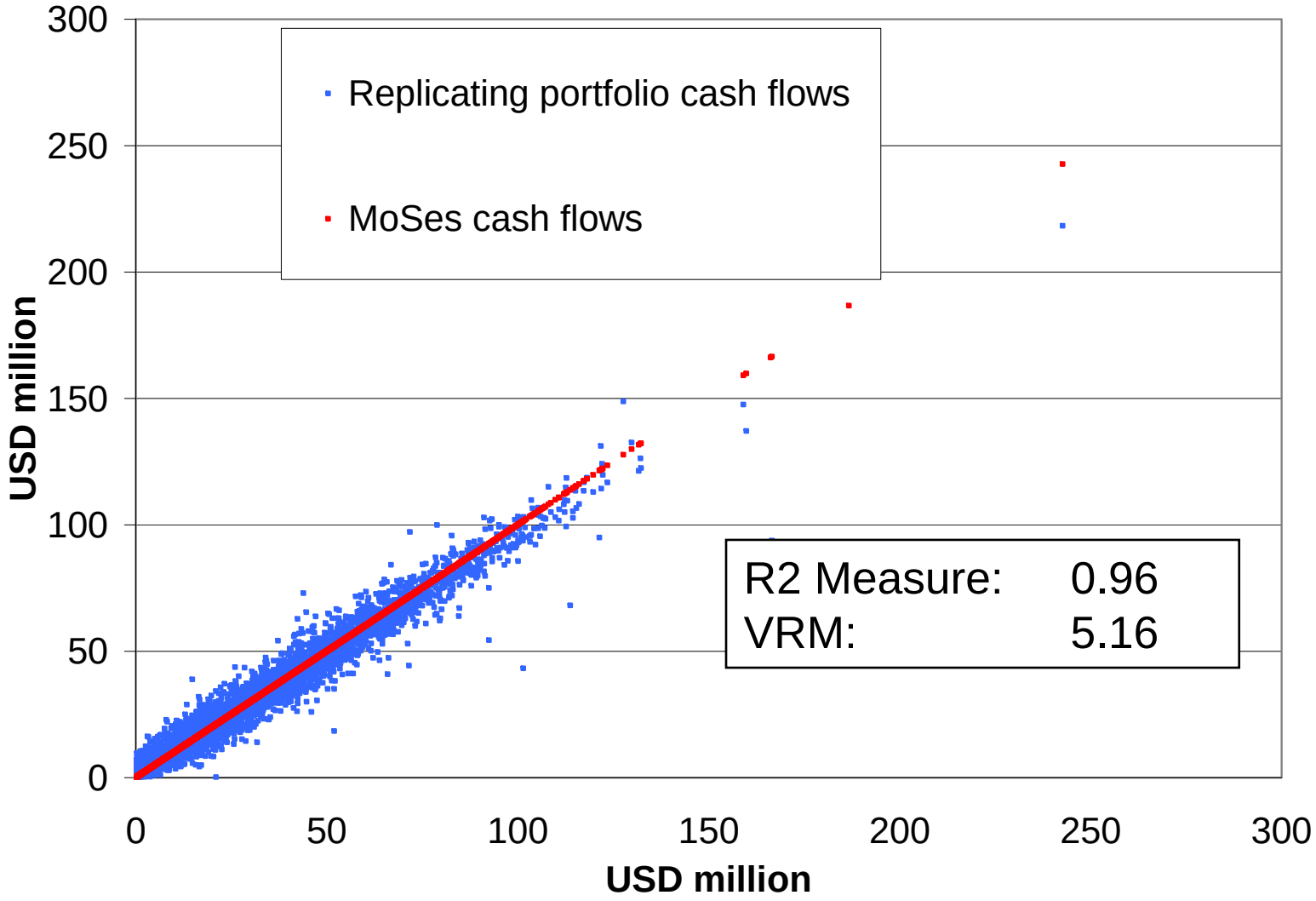
Replicating portfolio approach for VA dynamic hedging

- Numerical problems
  - The Greeks calculation usually requires repeated stochastic simulation for a large number of scenarios over a huge portfolio of contracts
    - time-consuming
    - numerical errors
  - Hedge effectiveness testing usually requires nested stochastic simulations
- Our research proves
  - The Replicating portfolio approach can be successfully used to fit VA business with
    - Complex path dependent policyholder behaviour
    - Complex guarantee and asset mix structure
  - The replicating assets are plain vanilla assets that will allow for
    - Faster and more accurate valuation
    - Hedging and market risk estimation, meaning derivation of Greeks
    - Hedge effectiveness testing without time consuming nested stochastic simulations

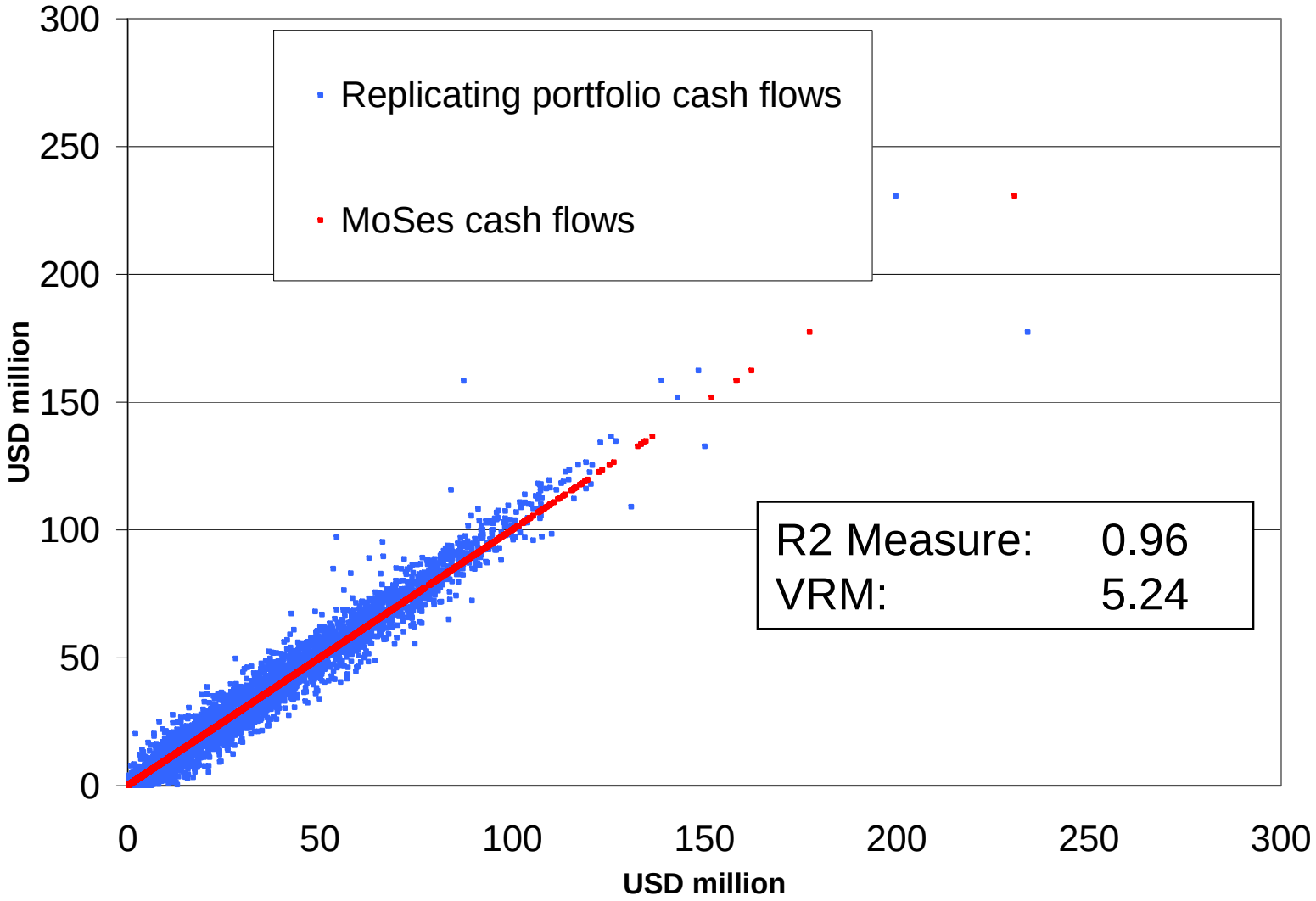
Replicating assets

- Replicating assets
  - Plain vanilla put options
  - Basket options (including the forward starting versions) on actual underlying
    - Simulating the asset mix of the underlying asset portfolio through combinations of 70-80% equity and 30-20% bonds
  - Knock-out basket options
    - When index level exceeds a certain level the option is knocked out, if the index never exceeds the knock-out level the option is in-force
- Closed forms or numerical approximations available

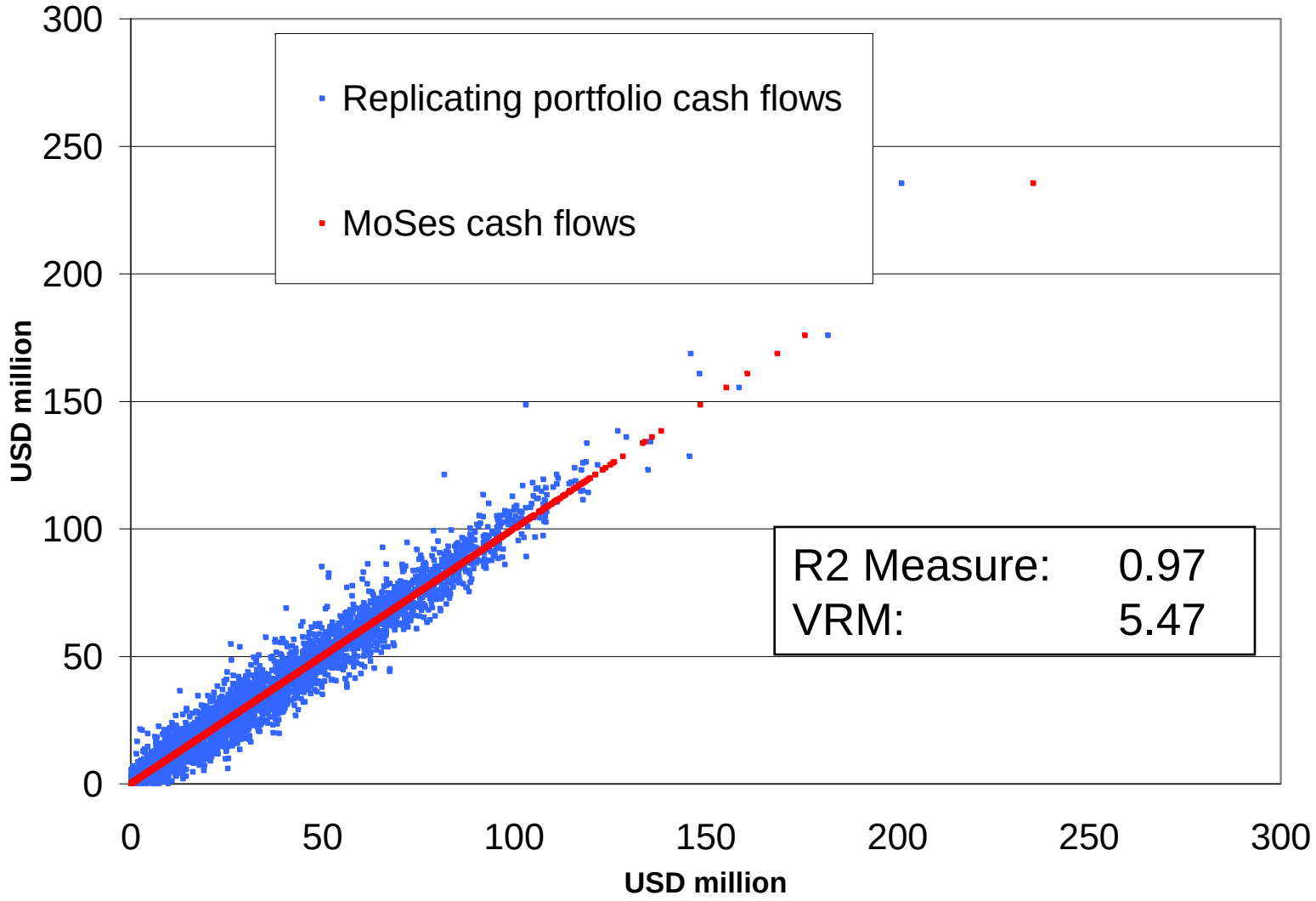
Results of central projection
Scatter plot: Central scenarios



Results of central projection
Scatter plot: Equity stress scenarios



Results of central projection
Scatter plot: Interest stress scenarios



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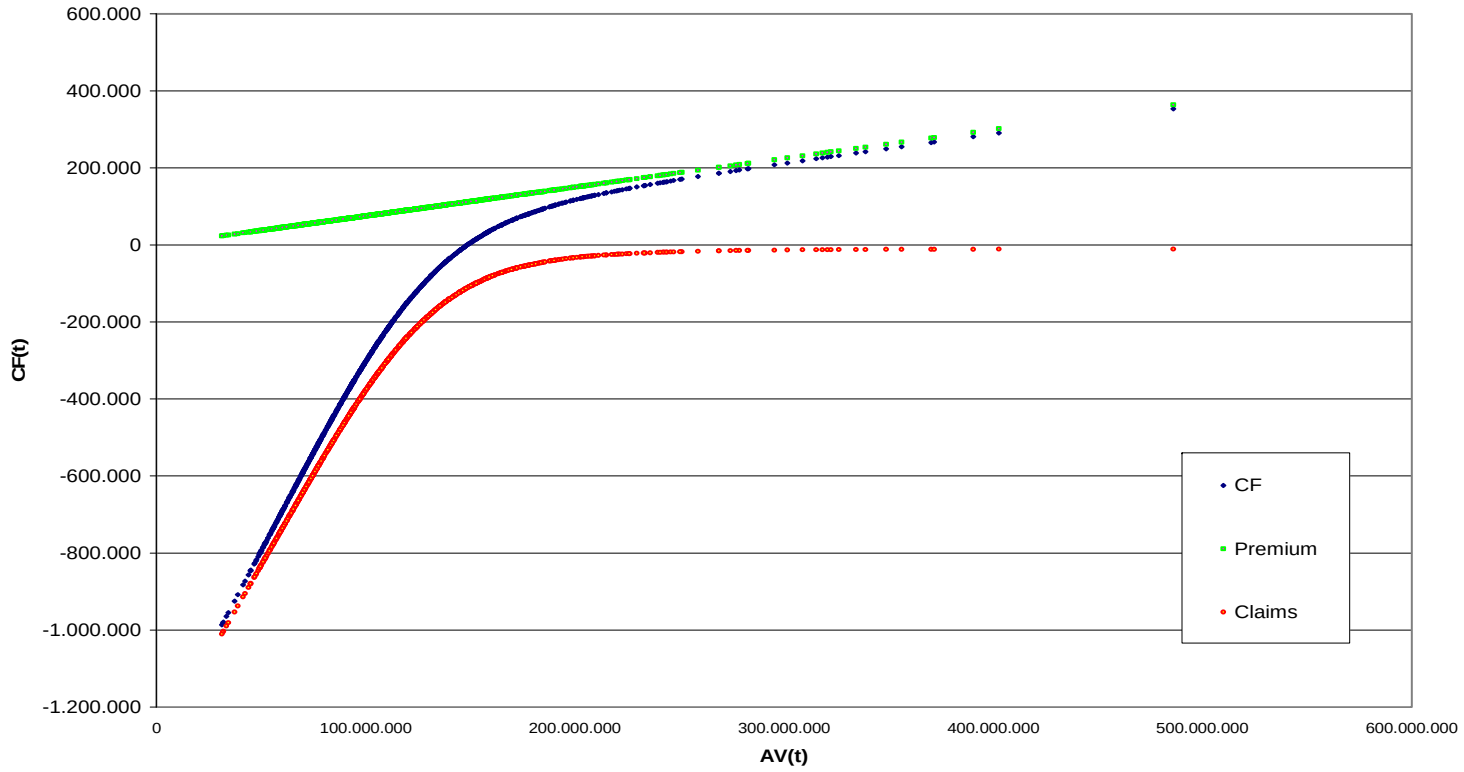
- SST target capital for a block of GMxBs – European reinsurer
- Typical German with profits business – the book value effect

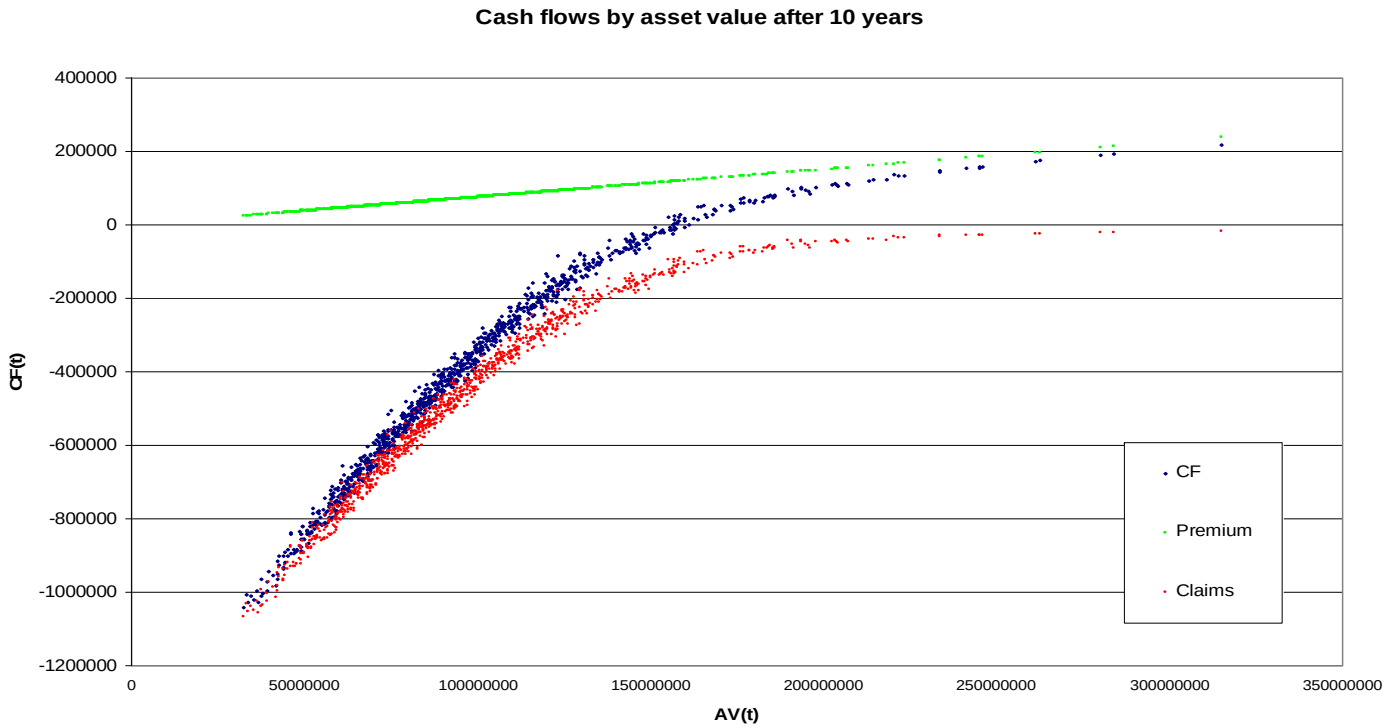
SST target capital for a block of GMxBs – European reinsurer

- Case study from 2005 (!)
- Includes policyholder behaviour (lapsation)
- Liabilities not straightforward: ratchets included
  - Thus the replication portfolio included floating strike discrete lookback options
    - Good approximation formula available
- Used for SST purposes
  - Valid approach as asymptotic behaviour is clear!

Plain vanilla contracts, no policyholder behaviour



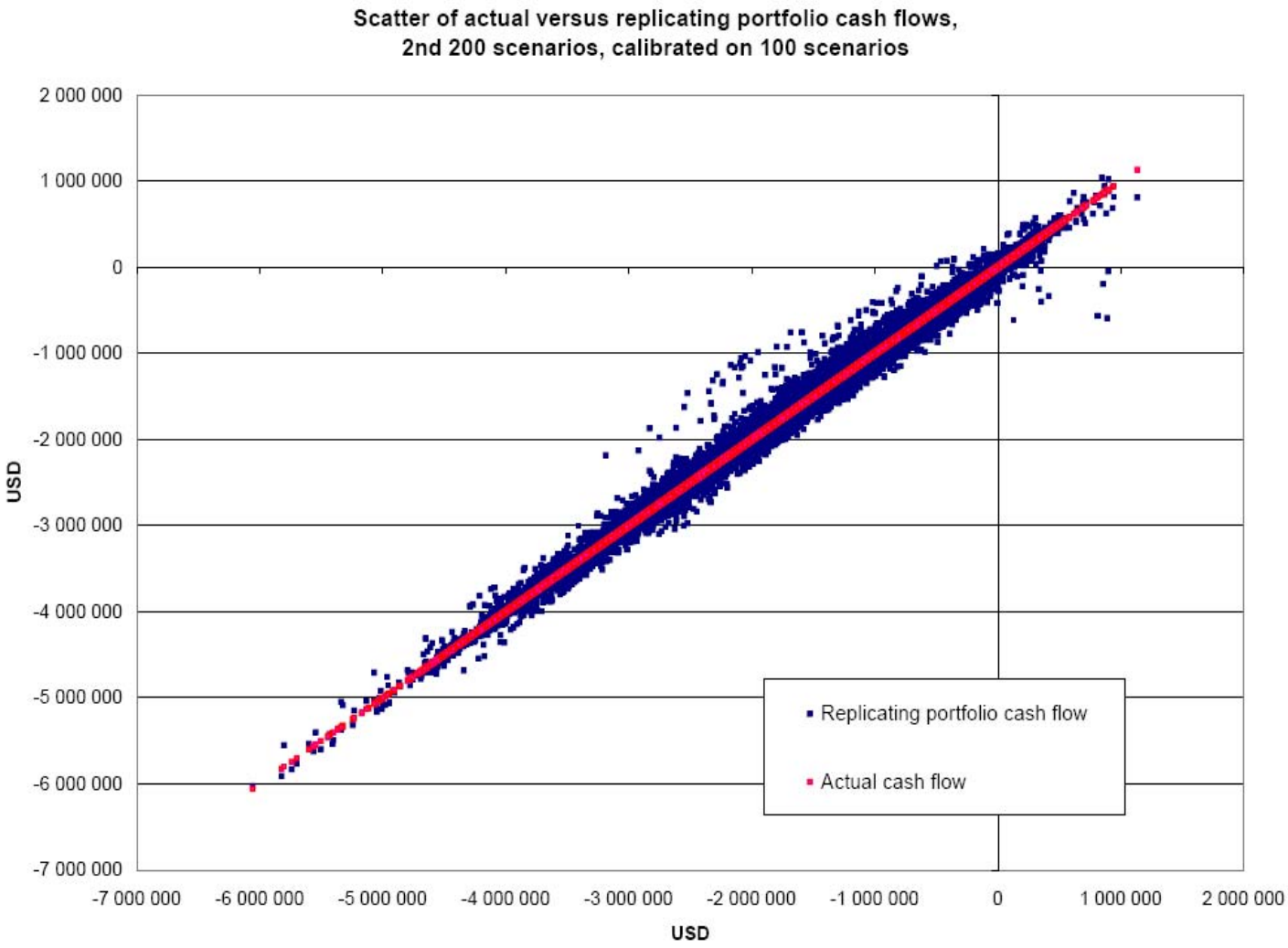
Plain vanilla contracts, with policyholder behaviour



The replication portfolio included floating strike discrete lookback options

		Number of assets purchased in each period									
		Quarter									
Assets	**Strike**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
European put 1	0.5	0	0	0	0	0	0	0	0	0	0
European put 2	0.8	0	0	0	0	0	0	0	0	-4'012'000	-4'012'000
European put 3	1.111	-653'000	-653'000	-653'000	-653'000	-2'825'000	-2'825'000	-2'825'000	-2'825'000	-2'351'000	-2'351'000
European put 4	1.667	-527'200	-527'200	-527'200	-527'200	-333'700	-333'700	-333'700	-333'700	-315'000	-315'000
European put 5	2.222	-472'900	-472'900	-472'900	-472'900	-301'500	-301'500	-301'500	-301'500	-285'100	-285'100
European put 6	2.778	-418'600	-418'600	-418'600	-418'600	-269'300	-269'300	-269'300	-269'300	-255'300	-255'300
European put 7	3.333	-364'300	-364'300	-364'300	-364'300	-237'100	-237'100	-237'100	-237'100	-225'400	-225'400
European put 8	3.889	-310'000	-310'000	-310'000	-310'000	-204'900	-204'900	-204'900	-204'900	-195'500	-195'500
European put 9	4.444	-255'700	-255'700	-255'700	-255'700	-172'600	-172'600	-172'600	-172'600	-165'700	-165'700
European put 10	10	287'300	287'300	287'300	287'300	149'500	149'500	149'500	149'500	133'000	133'000
Equity	N/A	690'100	690'100	690'100	690'100	430'400	430'400	430'400	430'400	404'600	404'600
Bond	N/A	97'740	97'740	97'740	97'740	57'990	57'990	57'990	57'990	53'760	53'760
Lookback put 1	0.5	-1'145'000	-1'145'000	-1'145'000	-1'145'000	-181'500	-181'500	-181'500	-181'500	-131'700	-131'700
Lookback put 2	0.8	-1'145'000	-1'145'000	-1'145'000	-1'145'000	-181'500	-181'500	-181'500	-181'500	-131'700	-131'700
Lookback put 3	1.111	-653'000	-653'000	-653'000	-653'000	-1'253'000	-1'253'000	-1'253'000	-1'253'000	-1'264'000	-1'264'000
Lookback put 4	1.667	-527'200	-527'200	-527'200	-527'200	-333'700	-333'700	-333'700	-333'700	-315'000	-315'000
Lookback put 5	2.222	-472'900	-472'900	-472'900	-472'900	-301'500	-301'500	-301'500	-301'500	-285'100	-285'100
Lookback put 6	2.778	-418'600	-418'600	-418'600	-418'600	-269'300	-269'300	-269'300	-269'300	-255'300	-255'300
Lookback put 7	3.333	-364'300	-364'300	-364'300	-364'300	-237'100	-237'100	-237'100	-237'100	-225'400	-225'400
Lookback put 8	3.889	-310'000	-310'000	-310'000	-310'000	-204'900	-204'900	-204'900	-204'900	-195'500	-195'500
Lookback put 9	4.444	-255'700	-255'700	-255'700	-255'700	-172'600	-172'600	-172'600	-172'600	-165'700	-165'700
Lookback put 10	10	287'300	287'300	287'300	287'300	149'500	149'500	149'500	149'500	133'000	133'000

Goodness of fit



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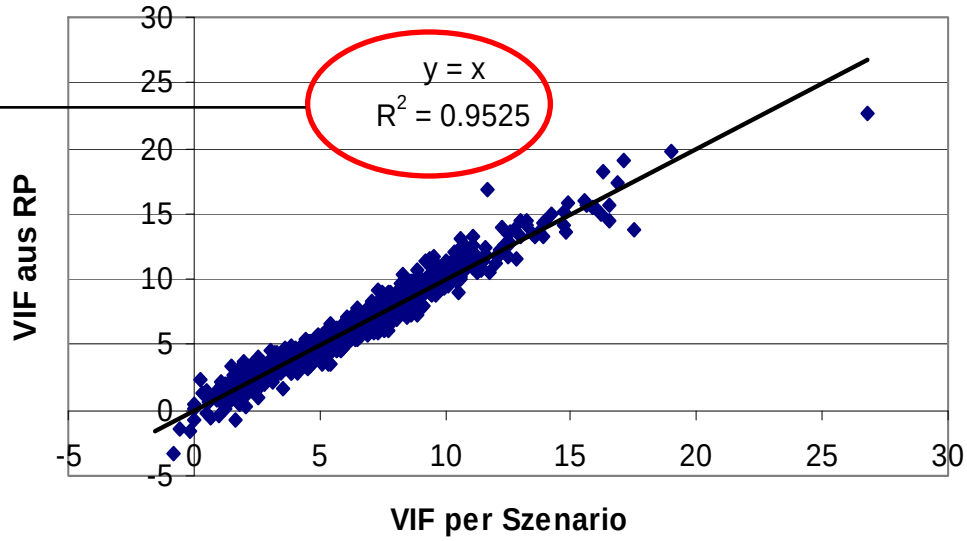
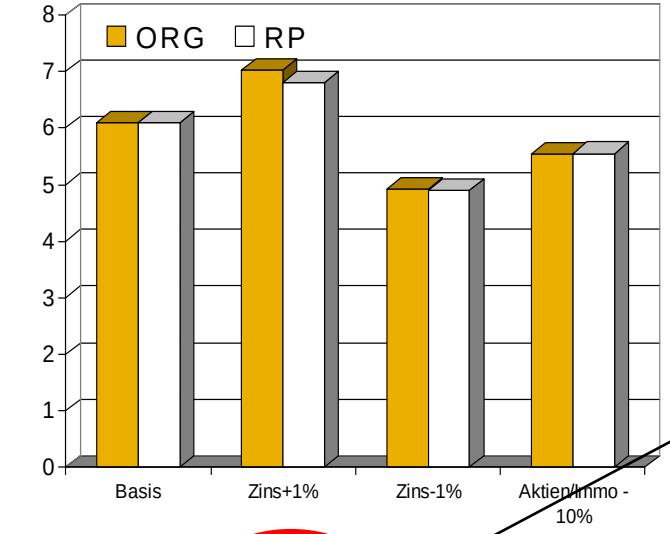
Fallstudie: RP für das deutsche gewinnberechtigte Geschäft

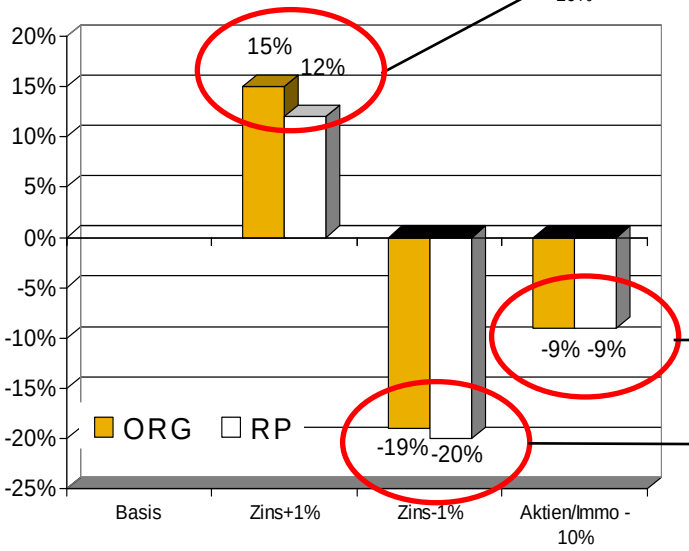
- Gesellschaft ABC schreibt hauptsächlich das typische deutsche gewinnberechtigte Geschäft: Kapital und Renten
- Für die Fallstudie wurden die Zahlen „anonymisiert“
- Approximation des VIFs unter MCEV

R^2 von 95% zeigt eine gute Anpassung des RP zu dem Dividenden-Cash Flow



Repricing: Um die Qualität zu verifizieren prüfen wir, ob wir mittels RP die ökonomischen Sensitivitäten replizieren können

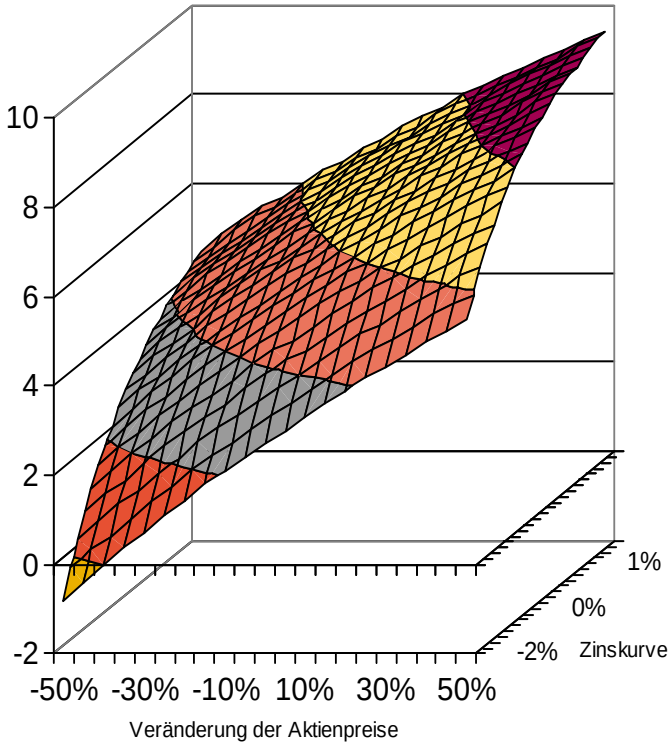


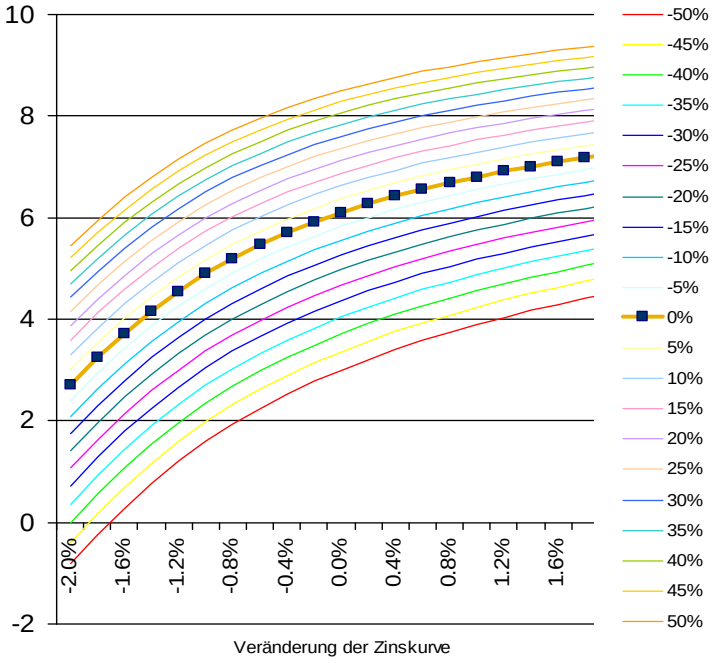


Die Sensitivität Zins+1% ist beim RP unterschätzt

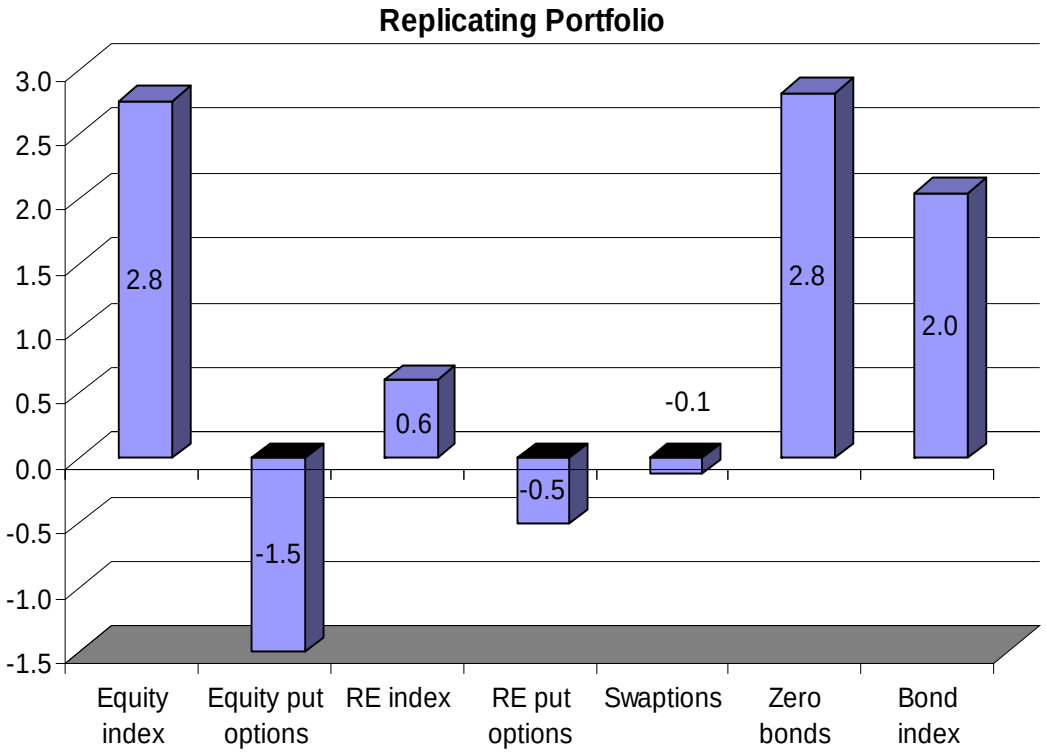
Das geschätzte RP repliziert sehr gut die Sensitivitäten: Aktien -10% und Zinsen -1%

Geschätztes RP erlaubt mühelos das ganze Spektrum der Sensitivitäten zu berechnen





RP spiegelt das Risiko des deutschen gewinnberechtigten Geschäftes wider



Illustrativ

- MCEV kann man mit „long“-Positionen in Aktien/Immobilien/Bonds und „short”-Positionen in Derivate replizieren
- Relativ niedriger Anteil der Swaptions liegt an „Moneyness“ der Swaptions (deep out-of-the-money)

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