“NatCat Models: Structure, Use and Implementation of the results into internal Solvency models”

SAV Conference, 2.9.2011
Irina Kaiser
Agenda

• Introduction
  – Definition of NatCat events
  – Use of the NatCat models
  – Insights into regulatory view
  – Importance of the NatCat models
  – Comparison of different NatCat Approaches

• Physical NatCat models
  – History of NatCat vendor models
  – Overview of existing NatCat vendors models
  – Structure of the models and approach of physical NatCat models
  – Output and Terminology of physical NatCat models
  – How to incorporate the results into the internal Solvency models
  – Advantages/ disadvantages in using the Physical NatCat models

• Future prospects

• Literature
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• **Future prospects**

• **Literature**
Swiss Re, 2011:

The term „natural catastrophe“ refers to an event caused by natural forces. Such an event generally results in a large number of individual losses involving many insurance policies. The scale of the losses resulting from a catastrophe depends not only on the severity of the natural forces concerned, but also on man-made factors, such as building design or the efficiency of disaster control in the affected region. Natural catastrophes are subdivided into the following categories: floods, storms, earthquakes, droughts/forest fires/heat waves, cold waves/frost, hail, tsunami and others natural catastrophes.
Introduction – Definition of NatCat events

Wikipedia:
A natural disaster is the effect of a natural hazard (e.g., flood, tornado, hurricane, volcanic eruption, earthquake or landslide). It leads to financial, environmental or human losses.

Munich Re, 2002:
A big natural catastrophe is defined as one where the affected region is “distinctly overtaxed, making interregional or international assistance necessary. This is usually the case when thousands of people are killed, hundreds of thousands are made homeless, or when a country suffers substantial economic losses, depending on the economic circumstances generally prevailing in that country”.
Introduction – Use of the NatCat models

• **Underwriting (insurer/reinsurer)**
  - Insurability of catastrophe risk / pricing of risk
  - Assessment of accumulation risk
  - Historical claims experiences are not sufficient to determine the tail of the distribution
  - Capital cost loadings
  - Determine how much coverage to buy (assisting the priority, plafond of the coverage, number of Ri`s, aggregated limits and deductibles)
  - To assess catastrophe risk and improve risk management decision

• **Reinsurance broker**
  - Run the models on client`s data and provide the output to interested reinsurer
  - Use the models as service to existing and potential clients in structuring and optimizing their reinsurance protection
  - Benchmarking of different NatCat vendor models

• **Capital market**
  - Pricing NatCat bonds
Introduction – Use of the NatCat models

• Regulatory solvency capital requirements
  – The results of physical NatCat models can be integrated into the spreadsheet of the standard model (partial internal models)
  – The results of physical NatCat models can be integrated into the internal model (full internal models)
  – Post-event estimates

• Rating agency
  – Require the results of the vendor models
  – S&P looks at 250-year PML aggregated over all perils (aggregated year view)
  – A.M.Best looks at greater 100-year PML Wind or 250-year PML Earthquake (occurrence view) (+ Stress Test: assume the first event modeled PML is fully paid from capital, assume the greatest 1 in 100 OEP modeled will occur as the second event)
Introduction – Insights into regulatory view

- **Regulatory capital requirements – Solvency II**
  - ensuring the capital requirements are calibrated at 99.5% 1-year Value at Risk

- **The CAT risk sub-module under the standard formula should be calculated using one of the following alternative methods (or as a combination of both):**
  - Method 1: standardized scenarios
  - Method 2: factor based methods

\[
NL\_CAT_1 = \text{The catastrophe capital requirement under method 1}
\]
\[
NL\_CAT_2 = \text{The catastrophe capital requirement under method 2}
\]
\[
NL\_CAT = \sqrt{NL\_CAT_1^2 + NL\_CAT_2^2}
\]

Source: Technical Specification
Introduction – Insights into regulatory view

• Regulatory capital requirements – Solvency II

– Method 1: standardized scenarios
  • Per peril and per country predefined scenarios for 1 in 200-year event (gross of RI)
  • Perils observed are: Windstorm, Flood, Earthquake, Hail, Subsidence
  • 1. Step: Calculate capital requirements at country level for each peril:

\[
CAT_{\text{peril}} = Q_{CTRY} \left( \sum_{r,c} AGG_{r,c}WTIV_{Zone,r}WTIF_{Zone,c} \right)
\]

– With WTIV as catastrophe charge for each cresta zone (Geographically weighted total insured value by zone):

\[
WTIV_{Zone} = F_{Zone}TIV_{Zone}
\]

– Q as 1 in 200 year factor for each country and peril
– AGG is an aggregation matrix by country
– F is relativity factors for each zone by country
– TIV is total insured value for Fire, Motor property damage (only for FL and H) and Marine (not for S) by zone
Introduction – Insights into regulatory view

• Regulatory capital requirements – Solvency II
  – Method 1: standardized scenarios
  • 2. Step: Aggregate capital requirements for each peril using correlation matrix over all countries:

\[ \text{CAT}_{\text{peril}} = \sqrt{\sum_{\text{ctry},i,i} \text{CORR}_{\text{ctry},i,j} \text{CAT}_{\text{peril}_{-\text{ctry},i}} \text{CAT}_{\text{peril}_{\text{ctry},j}}} \]

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• 3 Step: Aggregate capital requirements over all perils using correlation matrix:

\[ \text{NL}_{-\text{CAT}}_1 = \sqrt{\sum_{\text{ctry},i,i} \text{CORR}_{\text{peril},i,j} \text{CAT}_{\text{peril}_{i,j}} \text{CAT}_{\text{peril}_{i,j}}} \]

Source: Technical Specification
Introduction – Insights into regulatory view

• Regulatory capital requirements – Solvency II

– Method 2: factor based methods
  • Undertakings should apply the factor based method in circumstances such as:
    – When Method 1 is not appropriate
    – When partial internal model is not appropriate

\[
NL\_CAT_2 = \sqrt{\sum_{t=1,2,3,5} ((c_t P_t)^2 + (c_{11} P_{11}))^2 + \sum_{t=4,7,8,9,10,13} (c_t P_t)^2 + (c_6 P_6 + c_{12} P_{12})^2}
\]

– with P as estimate of the gross written premium during the forthcoming year in the relevant lines of business which are affected by the catastrophe event
– And C as the calibrated gross factors by event and applicable to all countries

<table>
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<tr>
<th>Events</th>
<th>Lines of business affected</th>
<th>Gross Factor ct</th>
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<td>Storm</td>
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<tr>
<td>Flood</td>
<td>Fire and property; Motor, other classes</td>
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<tr>
<td>Earthquake</td>
<td>Fire and property; Motor, other classes</td>
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<tr>
<td>Hail</td>
<td>Motor, other classes</td>
<td>30%</td>
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</table>

Source: Technical Specification

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Introduction – Insights into regulatory view

• Results of QIS 5 Study:
  – NatCat Risk accounts for 42% of the total non-life UW risk
  – Premium & Reserve risk account for 58%
  – Diversification effect is 22%

Source: EIOPA Report on the fifth Quantitative Impact Study (QIS5) for Solvency II, 14.03.2011
Introduction – Insights into regulatory view

• Results on QIS 5 Study:
  – Split of the NatCat Risk per peril:
    • Windstorm: 39%
    • Flood: 13%
    • Earthquake: 34%
    • Hail: 12%
    • Subsidence: 1%
    • Diversification: 38%

Source: EIOPA Report on the fifth Quantitative Impact Study (QIS5) for Solvency II, 14.03.2011
Introduction – Insights into regulatory view

• Regulatory capital requirements – SST
  – ensuring the capital requirements are calibrated at 99.0% 1-year Expected Shortfall (TVaR))
  
• Natural hazard pool modelled as the market-wide claims, then the market share is applied
  – Major claims are modelled with
    • Generalized Pareto distribution with threshold= CHF 50m /a=1.25 und b=18.8
    • Max claim for the market-wide event is CHF 500m
    • Frequency: Poisson with lambda = 0.67

• Other natural hazards modelled (business interruption):
  – 20% of the NHP claims for property (fully correlate with the NHP Claims)
  – Max claim for the market-wide event is market share × CHF 1bn

• Modelling of cumulated claims due to hail events for comprehensive motor vehicle insurance
  – Pareto distribution for claim size with alpha=1,85 and threshold= CHF 45m for the market-wide event
  – Poisson distribution for the frequency of claims with lambda=0.9 for the market-wide event

Source: Technical document on the Swiss Solvency Test
Introduction – Importance of NatCat model

• Increased risk for Insurance Industry / losses out of catastrophic events
Introduction – Importance of NatCat model

• Increased risk for Insurance Industry / number of catastrophic events

Source: Sigma1_2011
Introduction - Comparison of different NatCat modeling Approaches

- **Physical NatCat models (exposure based, synthetic event catalogue)**
  - High time/space required for computing
  - Scientific view on vulnerability from many perspectives
  - Modeling insights: Black Box Character

- **Stochastic NatCat models (historical experience based)**
  - Only possible having long time observations
  - Usually not sufficient data available (low/no data for earthquake)
  - High transparency

- **Zonal Systems**
  - Distribution of the risks into different vulnerability zones
  - Used in pricing/rating
  - What is the size of PML?

- **Scenario-based approach**
  - QIS5
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- **Future prospects**

- **Literature**
History of NatCat vendors models

- **Property Insurance**
  - Mapping the risk on a wall-hang map
  - 1800-1960
  - Development of GIS Systems

- **Natural Hazard Science**
  - Understanding nature and impact of natural hazards (measuring hazard intensity)
  - 1800-seismograph, anemometer
  - 1970- study about the frequency of NatCat events

- **Computer-based models**
  - Provide estimates of NatCat losses by overlapping the property at risk with the potential natural hazard sources in the geographical area
  - AIR(87), RMS(88), EQE(94)

- 21.09.1989 – Hurricane Hugo, 4 billion insurance loss (South Carolina)
- 17.10.1989 – Earthquake Loma Prieta, 6 billion insurance loss (San Francisco)
- 08.1992 – Hurricane Andrew, 15.5 billion insurance loss (Florida), AIR est. 13 bn.
  - (9 insurance become insolvent)
  - Need to estimate NatCat risk more precisely

1997 – HAZUS – open source FEMA model to assess EQ Risk in US

2004 – HAZUS-MH – included Wind and Flood
Overview of existing NatCat vendors models

• 
  Risk Management Solutions (RMS) was formed in 1988 at Stanford University
  – RMS models risk in over 100 countries, allowing stakeholders to analyze the probability of loss in regions with the highest exposure. Our models are built using detailed data reflecting highly localized variations in hazard, and databases capturing property and human exposures. They are continually updated with the latest scientific research and data.

  – Through its modeling software platform, WORLDCATenterprise™, EQECAT enables clients to quantify and manage the potential financial impact of natural hazards. WORLDCATenterprise™ includes 181 natural hazard software models for 95 countries spanning six continents. These models are based upon innovative applications of the latest science, engineering expertise, claims and exposure data and advanced mathematics.
Overview of existing NatCat vendors models

• **AIR Worldwide** was founded in **1987 in Boston**
  - AIR Worldwide (AIR) is the scientific leader and most respected provider of risk modelling software and consulting services. AIR founded the catastrophe modelling industry **in 1987** and today models the **risk from natural catastrophes and terrorism in more than 90 countries**. More than **400 insurance, reinsurance, financial, corporate, and government clients rely on AIR software and services for catastrophe risk management, insurance-linked securities, detailed site-specific wind and seismic engineering analyses, agricultural risk management, and property replacement-cost valuation**. AIR is a **member of the Verisk Insurance Solutions group** at **Verisk Analytics** and is headquartered in Boston with additional offices in North America, Europe, and Asia.
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• **Literature**
Structure of physical NatCat models

- The four basic components are:

  Where to happen?  
  At which intensity?  
  How often?

  → To which damage degree?  How high is the loss?

  Where the risks are?

  Hazard

  Value Distribution

  Vulnerability

  Loss
Structure of physical NatCat models

- Hazard module:

  - Historical event catalogue
    - (Lothar, Martin, ...)
  - Stochastical sampling
    - (Wind direction, Wind duration, ...)
  - Complete catalogue of syntetical storm events
Structure of physical NatCat models

- **Value Distribution module:**
  - Per each location (Cresta Zone, Post Code, Street Level, latitude/longitude)
  - Sum Insured / Number Risks
  - Currency
  - Risk Type (Residential, Commercial, Agricultural, Industrial)
  - Coverage (Building, Contents, BI)
  - Peril covered (Windstorm, Flood, Earthquake, Hail)

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<th>LOCATOR</th>
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Structure of physical NatCat models

• Vulnerability module:

  – The vulnerability function converts the intensity of the event (magnitude, water height, wind speed) into loss amounts for each object within a calculation unit
  – Different vulnerability functions exist for different occupancy type: Residential, Commercial, Industrial and Agricultural
  – Further refinement can be done due to e.g. the age and construction type of the building (secondary modifiers)
  – Calibrated using historical loss experience, engineering analysis of building types and expert judgement
Structure of physical NatCat models

- **Loss module:**
  - The damage ratio distribution for a specific event is then multiplied by the building replacement value to obtain the loss per event.
  - Here insurance policy conditions like limits, deductibles and reinsurance terms are incorporated.
  - The combined loss distribution of all buildings is computed using convolution process.
  - Synthetic loss history of x.000 Years is simulated.
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Output and Terminology of physical NatCat models

- Exceedance probability curves (EP Curve):
  - Annual Exceedance probability curve (AEP) – aggregated annual loss
  - Occurrence Exceedance probability curve (OEP) – max occurrence loss

\[ P(\text{Loss}>7\text{m})=0.3\% \]

Loss of 7m equates to 0.3% percentile of loss distribution or loss of 7m has a return period of \( \frac{1}{0.003}=333 \) years
Output and Terminology of physical NatCat models

Output: Event by event results - EQECAT

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<th>LOCATION</th>
<th>SCENARIO</th>
<th>TITLE</th>
<th>FREQUENCY</th>
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<td>3,658,642,106,949</td>
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3,006 events in total.
### Output and Terminology of physical NatCat models

#### Output: Event by Event results - RMS

<table>
<thead>
<tr>
<th>Event ID</th>
<th>Gross Loss</th>
<th>Std. Deviation (Independent)</th>
<th>Std Deviation (Correlated)</th>
<th>Exposure Value</th>
<th>Annual Rate</th>
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<tr>
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<td>1,543,574,199,604</td>
<td>0.0009476</td>
</tr>
<tr>
<td>97766</td>
<td>768,981,532,454</td>
<td>60,572,602,523</td>
<td>69,447,871,541</td>
<td>1,543,574,199,604</td>
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<td>57,135,053,483</td>
<td>1,778,456,278,131</td>
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<tr>
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</table>

1,650 EQ events in RMS model event set
Output and Terminology of physical NatCat models

Output: Event by Event results - AIR

<table>
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<th>* Year</th>
<th>Event C</th>
<th>Total.GU</th>
<th>Total.GR</th>
<th>Total.NT</th>
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<td>1091451533</td>
<td>1091451533</td>
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</tr>
</tbody>
</table>
Agenda

• Introduction
  – Definition of NatCat events
  – Use of the NatCat models
  – Insights into regulatory view
  – Importance of the NatCat models
  – Comparison of different NatCat Approaches

• Physical NatCat models
  – History of NatCat vendor models
  – Overview of existing NatCat vendors models
  – Structure of the models or Approach of physical NatCat models
  – Output and Terminology of physical NatCat models
  – How to incorporate the results into the internal Solvency models
  – Advantages/ disadvantages in using the Physical NatCat models

• Future prospects

• Literature
How to incorporate the results into the internal solvency models

Event Loss Set:

- Each row of the ELS represents a collective risk model with the parameter $\lambda_i$ for the Frequency distribution of the event $i$, $i=1,..,n$ and the parameter $\alpha_i$ und $\beta_i$ for the Size distribution of the event $i$, $i=1,..,n$. ($n$ – is the number of rows or the number of events in the ELS)

- Under the assumption of Poisson distribution for the Frequency $N_i$ for the Event $i$, is the total loss distribution for the Event $i$, $i=1,..,n$:

$$S_i = X_{i,1} + X_{i,2} + \cdots + X_{i,N_i} \sim P_i = \sum_{k=0}^{\infty} R_i\{k\}Q_i^k$$

- Loss Number $N_i$ is Poisson distributed with the Parameter $\lambda_i$
- Loss Size $X_{i,1}, X_{i,2}, \ldots, X_{i,N_i} \sim Q_i$ is Beta distributed with the Parameters $\alpha_i$ und $\beta_i$
How to incorporate the results into the internal solvency models

Theorem:

• Are $N_1, \ldots, N_n$ stochastically Independent Poisson distributed random variables with the parameters $\lambda_1, \ldots, \lambda_n > 0$ and $X_{i,j}$ with $0 \leq i \leq n, j \in \mathbb{Z}_+$ positive independent also of $N_i$ as $Q_i$ distributed random variables.

• Then is the distribution of

$$S = \sum_{i=1}^{n} \sum_{j=1}^{N_i} X_{i,j} \sim P = P_1 \ast \cdots \ast P_n = PSV(\lambda, Q)$$

• With $\lambda_1 + \cdots + \lambda_n = \lambda$ and $Q = \sum_{i=1}^{n} \frac{\lambda_i}{\lambda} Q_i$

• $n$ – is the number of rows in the ELS or number of Event in the ELS
How to incorporate the results into the internal solvency models

With all these assumption can AEP and OEP curve be directly calculated using the ELS

\[ M := \max \{X_{i,j}, 1 \leq i \leq n, 1 \leq j \leq N_i\}, S := \sum_{i=1}^{n} \sum_{j=1}^{N_i} X_{i,j} \]

- OEP Curve – can be calculated directly from the ELS

\[ P(M > x) = 1 - \exp \left\{-\lambda [1 - Q(x)]\right\} \quad \text{(OEP-Kurve)} \]

- AEP Curve – Simulation, Panjer Algorithms, FFT

\[ P(S > x) = 1 - e^{-\lambda} - e^{-2} \sum_{i=1}^{n} \frac{\lambda^i}{i!} Q^{*i} (x) \quad \text{(AEP-Kurve)} \]

- With \( \lambda_1 + \cdots \lambda_n = \lambda \) and \( Q = \sum_{i=1}^{n} \frac{\lambda_1}{\lambda} Q_i \)

- \( n \) – ist die Anzahl der Zeilen oder Anzahl der Ereignisse im ELS
How to incorporate the results into the internal solvency models

1. Option

Event Loss Set → e.g. ReMetrica / Igloo

2. Option

Event Loss Set → In-house internal model / Excel (VBA)

3. Option

Frequency / Severity Distribution → In-house internal model / Excel (VBA)
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Advantages/Disadvantages in using the physical NatCat models

• **Advantages:**
  – The results can be easily incorporated into the internal solvency models using Event Loss Table Outputs (ELT)
  – The results are accepted and required by the rating agency

• **Disadvantages:**
  – Model changes
  – Black Box Character
  – Model understanding
  – Model uncertainty
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• Future prospects

• Literature
Future Prospects

• „Open source“ models, jointly developed by scientists, engineers and industry sectors for each single risk
Literatur

P. Grosssi, H. Kunreuther, „Catastrophy Modelling: A new approach to managing risk“
W. Dong, „Modern Portfolio Theory with Application to Catastrophe Insurance“ (RMS)
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Thank you very much for your Attention