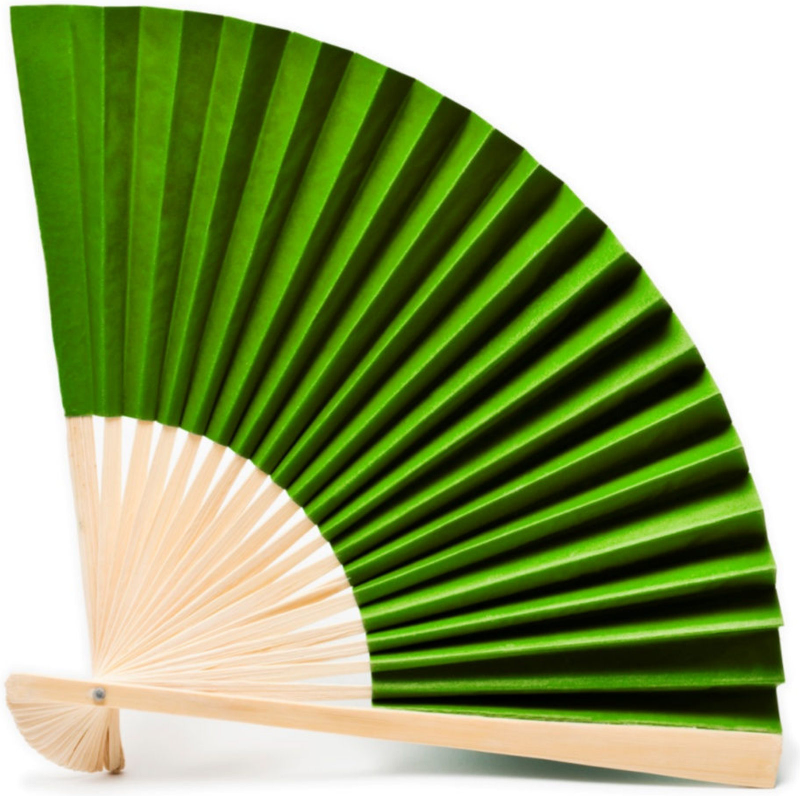
Deloitte.

„NatCat Models: Structure, Use and Implementation of the results into internal Solvency models “



SAV Conference, 2.9.2011

Irina Kaiser

Audit.Tax.Consulting.Corporate Finance.

Agenda

- **Introduction**
  - – Definition of NatCat events
  - – Use of the NatCat models
  - – Insights into regulatory view
  - – Importance of the NatCat models
  - – Comparison of different NatCat Approaches

- **Physical NatCat models**
  - – History of NatCat vendor models
  - – Overview of existing NatCat vendors models
  - – Structure of the models and approach of physical NatCat models
  - – Output and Terminology of physical NatCat models
  - – How to incorporate the results into the internal Solvency models
  - – Advantages/ disadvantages in using the Physical NatCat models

- **Future prospects**

- **Literature**

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Introduction – Definition of NatCat events

Swiss Re, 2011:

The term **„natural catastrophe"** refers to an **event caused by natural forces**. Such an event generally results in a **large number of individual losses** involving **many insurance policies**. The **scale of the losses** resulting from a catastrophe **depends** not only on the **severity of the natural forces concerned**, but also on **man-made factors**, such as **building design** or the **efficiency of disaster control** in the affected region. Natural catastrophes are subdivided into the following categories**: floods, storms, earthquakes, droughts/forest fires/heat waves, cold waves/frost, hail, tsunami and others natural catastrophes.**

Introduction – Definition of NatCat events

Wikipedia:

A natural disaster is the effect of a **natural hazard** (e.g., **flood**, **tornado**, **hurricane**, **volcanic eruption**, **earthquake or landslide**). It leads **to financial**, **environmental or human losses**.

Munich Re, 2002:

A big **natural catastrophe** is defined as one

where the affected region is "distinctly overtaxed, making interregional or international assistance necessary. This is usually the case when **thousands of people are killed, hundreds of thousands are made homeless**, **or** when a **country suffers substantial economic losses**, depending on the economic circumstances generally prevailing in that country" .

Introduction – Use of the NatCat models

- **Underwriting (insurer/reinsurer)**
  - Insurability of catastrophe risk / pricing of risk
  - Assessment of accumulation risk
  - Historical claims experiences are not sufficient to determine the tail of the distribution
  - Capital cost loadings
  - Determine how much coverage to buy (assisting the priority, plafond of the coverage, number of Ri`s, aggregated limits and deductibles)
  - To assess catastrophe risk and improve risk management decision
- **Reinsurance broker**
  - Run the models on client`s data and provide the output to interested reinsurer
  - Use the models as service to existing and potential clients in structuring and optimizing their reinsurance protection
  - Benchmarking of different NatCat vendor models
- **Capital market**
  - Pricing NatCat bonds

Introduction – Use of the NatCat models

- **Regulatory solvency capital requirements**
  - The results of physical NatCat models can be integrated into the spreadsheet of the standard model (partial internal models)
  - The results of physical NatCat models can be integrated into the internal model (full internal models)
  - Post-event estimates
- **Rating agency**
  - Require the results of the vendor models
  - S&P looks at 250-year PML aggregated over all perils (aggregated year view)
  - A.M.Best looks at greater 100-year PML Wind or 250-year PML Earthquake (occurrence view) (+ Stress Test: assume the first event modeled PML is fully paid from capital, assume the greatest 1 in 100 OEP modeled will occur as the second event)

Introduction – Insights into regulatory view

- **Regulatory capital requirements – Solvency II**
  - ensuring the capital requirements are calibrated at 99.5% 1-year Value at Risk

- **The CAT risk sub-module under the standard formula should be calculated using one of the following alternative methods (or as a combination of both):**
  - Method 1: standardized scenarios
  - Method 2: factor based methods

NL_CAT_1 = The catastrophe capital requirement under method 1

NL_CAT_2 = The catastrophe capital requirement under method 2

$$NL\_CAT = \sqrt{NL\_CAT_1^2 + NL\_CAT_2^2}$$

Source: Technical Specification

Introduction – Insights into regulatory view

- **Regulatory capital requirements – Solvency II**

  – **Method 1: standardized scenarios**
  - Per peril and per country predefined scenarios for 1 in 200- year event (gross of RI)
  - Perils observed are: Windstorm, Flood, Earthquake, Hail, Subsidence
  - 1. Step: Calculate capital requirements at country level for each peril:

$$CAT_{peril\_ctry} = Q_{CTRY} \sqrt{\sum_{r,c} AGG_{r,c} WTIV_{Zone,r} WTIF_{Zone,c}}$$

  – With WTIV as catastrophe charge for each cresta zone (Geographically weighted total insured value by zone):

$$WTIV_{Zone} = F_{Zone} TIV_{Zone}$$

  – Q as 1 in 200 year factor for each country and peril
  – AGG is an aggregation matrix by country
  – F is relativity factors for each zone by country
  – TIV is total insured value for Fire, Motor property damage (only for FL and H) and Marine (not for S) by zone

Source: Technical Specification

Introduction – Insights into regulatory view

- **Regulatory capital requirements – Solvency II**
  - **Method 1: standardized scenarios**
    - 2. Step: Aggregate capital requirements for each peril using correlation matrix over all countries:

$$CAT_{peril} = \sqrt{\sum_{ctry,i,i} CORR_{ctry,i,j} CAT_{peri\_ctry,i} CAT_{peril\_ctry,j}}$$

Hail Correlations									
	AT	BE	FR	DE	IT	LU	NL	CH	ES
AT	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
BE	0.00	1.00	0.25	0.00	0.00	0.50	0.50	0.00	0.00
FR	0.00	0.25	1.00	0.00	0.00	0.25	0.00	0.25	0.00
DE	0.00	0.00	0.00	1.00	0.00	0.25	0.25	0.00	0.00
IT	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00
LU	0.00	0.50	0.25	0.25	0.00	1.00	0.00	0.00	0.00
NL	0.00	0.50	0.00	0.25	0.00	0.00	1.00	0.00	0.00
CH	0.00	0.00	0.25	0.00	0.00	0.00	0.00	1.00	0.00
ES	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00

    - 3 Step: Aggregate capital requirements over all perils using correlation matrix:

	Windstorm	Earthquake	Flood	Hail	Subsidence
Windstorm	1.00	0.00	0.25	0.25	0.00
Earthquake	0.00	1.00	0.00	0.00	0.00
Flood	0.25	0.00	1.00	0.00	0.00
Hail	0.25	0.00	0.00	1.00	0.00
Subsidence	0.00	0.00	0.00	0.00	1.00

$$NL\_CAT_1 = \sqrt{\sum_{ctry,i,i} CORR_{peril,i,j} CAT_{peril,i,j} CAT_{peril,i,j}}$$

Source: Technical Specification

Introduction – Insights into regulatory view

- **Regulatory capital requirements – Solvency II**

  – **Method 2: factor based methods**

  - Undertakings should apply the factor based method in circumstances such as:
    - – When Method 1 is not appropriate
    - – When partial internal model is not appropriate

$$NL\_CAT_2 = \sqrt{\sum_{t=1,2,3,5} ((c_t P_t)^2 + (c_{11} P_{11}))^2 + \sum_{t=4,7,8,9,10,13} (c_t P_t)^2 + (c_6 P_6 + c_{12} P_{12})^2}$$

– with P as estimate of the gross written premium during the forthcoming year in the relevant lines of business which are affected by the catastrophe event

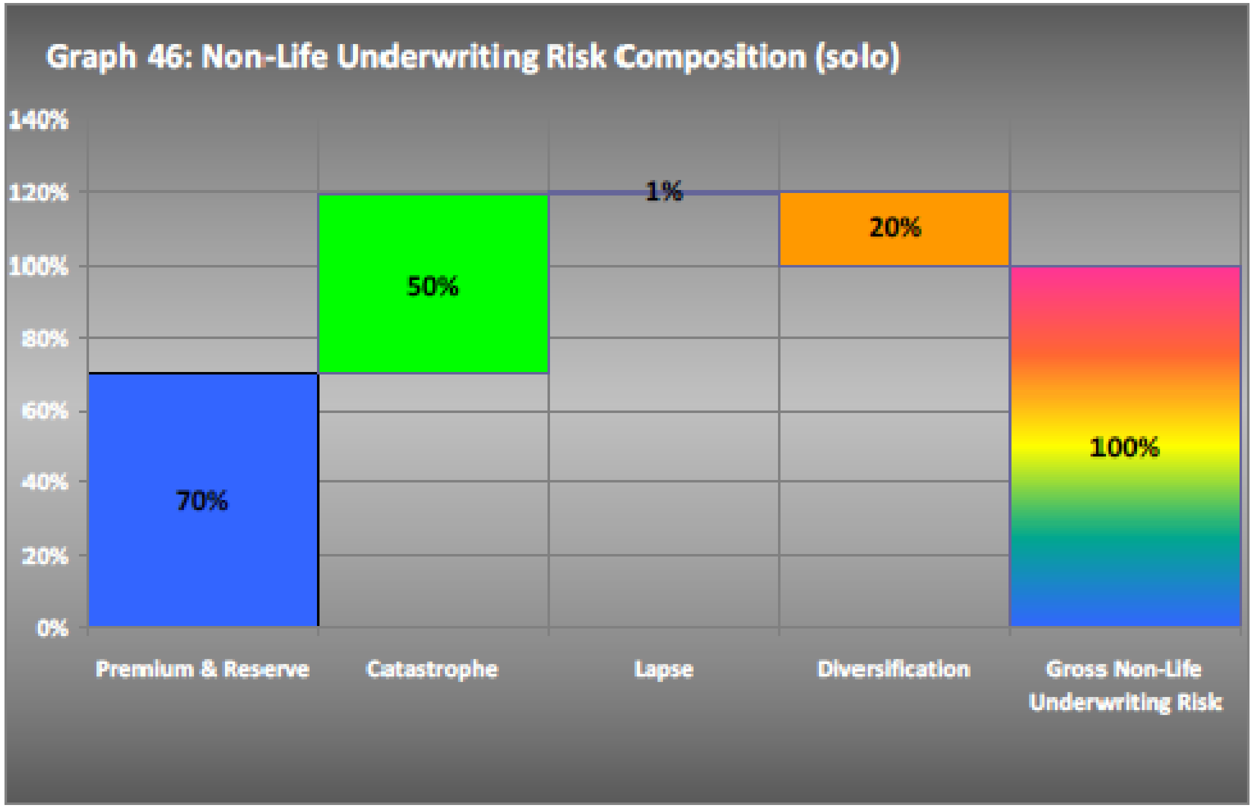
– And C as the calibrated gross factors by event and applicable to all countries

Events	Lines of business affected	Gross Factor ct
Storm	Fire and property; Motor, other classes	175%
Flood	Fire and property; Motor, other classes	113%
Earthquake	Fire and property; Motor, other classes	120%
Hail	Motor, other classes	30%

Source: Technical Specification

Introduction – Insights into regulatory view

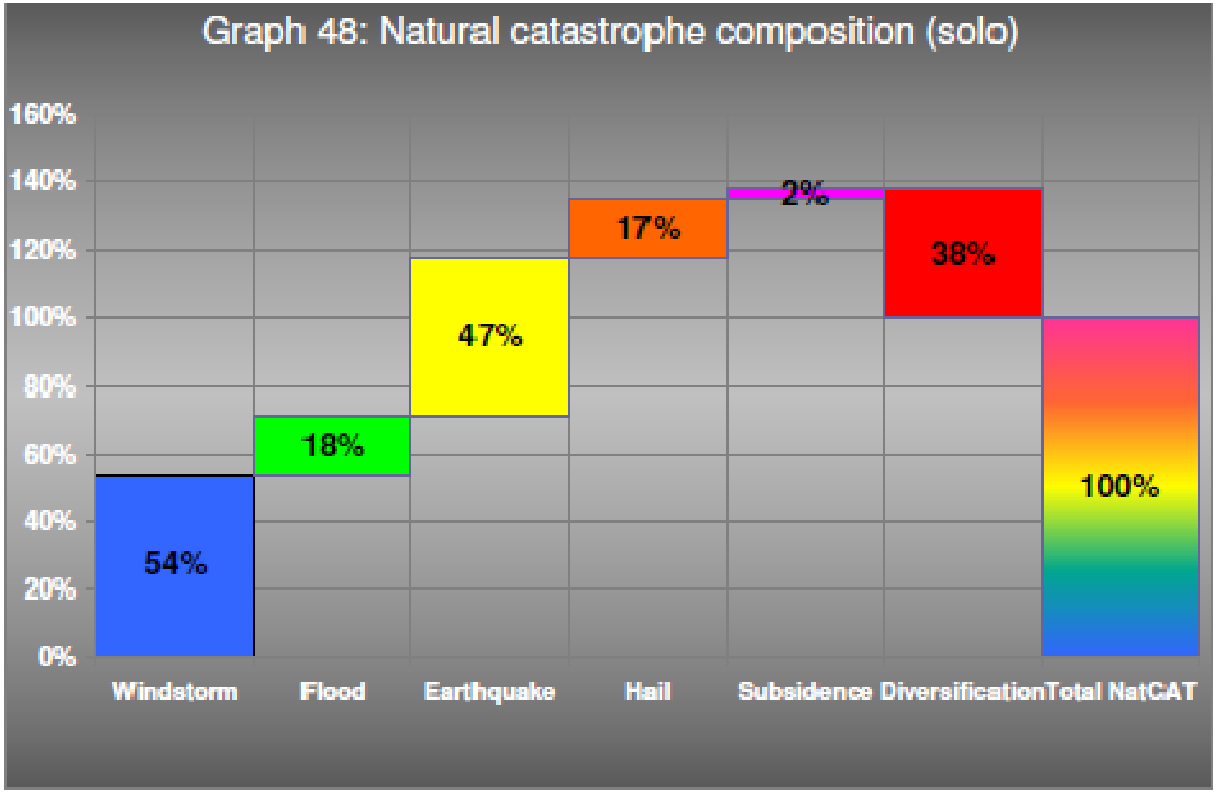
- **Results of QIS 5 Study:**
  - – NatCat Risk accounts for 42% of the total non-life UW risk
  - – Premium & Reserve risk account for 58%
  - – Diversification effect is 22%



Source: EIOPA Report on the fifth Quantitative Impact Study (QIS5) for Solvency II, 14.03.2011

Introduction – Insights into regulatory view

- **Results on QIS 5 Study:**
  – Split of the NatCat Risk per peril:
    - Windstorm: 39%
    - Flood: 13%
    - Earthquake: 34%
    - Hail: 12%
    - Subsidence: 1%
    - Diversification: 38%



Source: EIOPA Report on the fifth Quantitative Impact Study (QIS5) for Solvency II, 14.03.2011

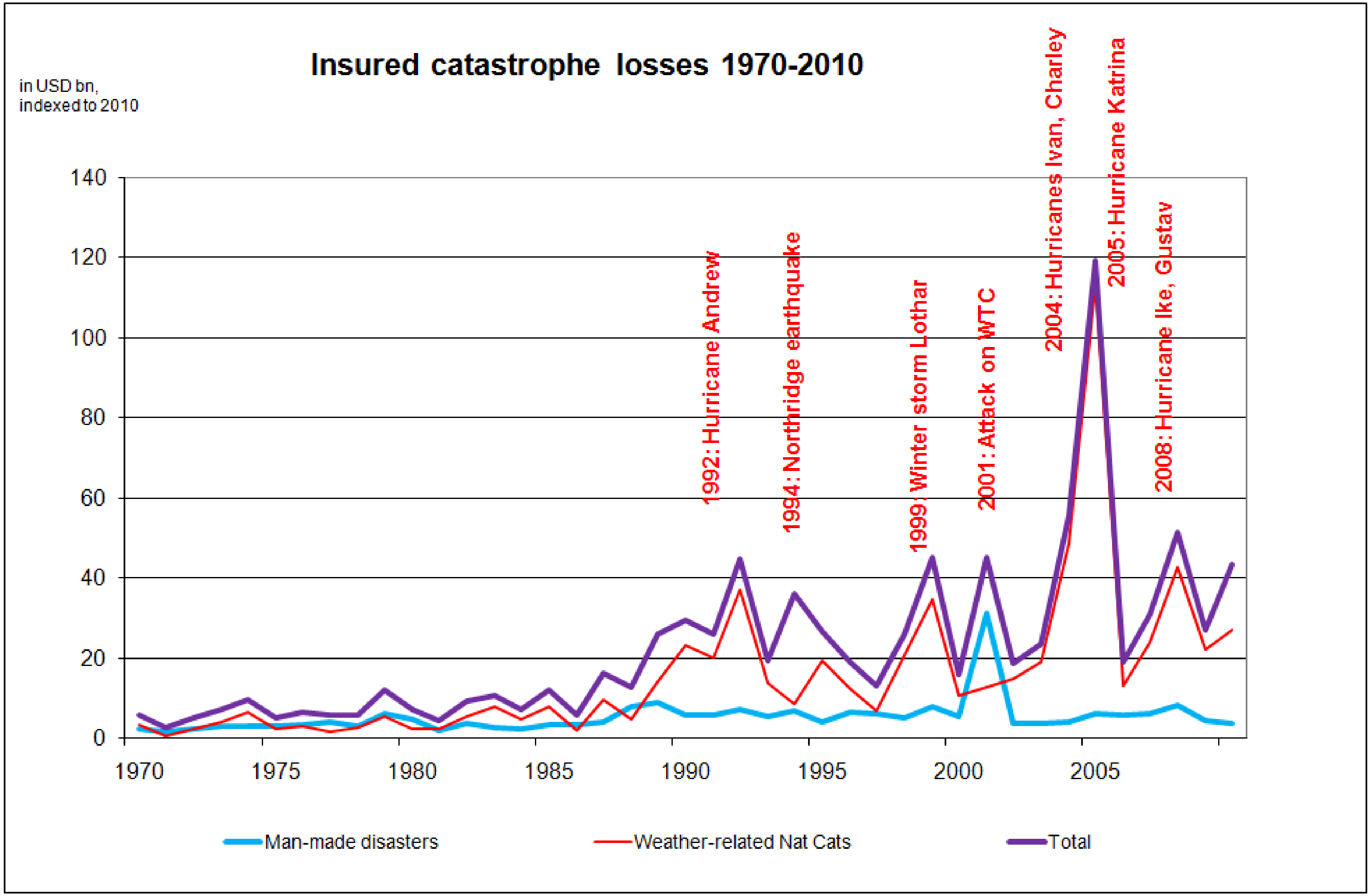
Introduction – Insights into regulatory view

- **Regulatory capital requirements – SST**
  - ensuring the capital requirements are calibrated at 99.0% 1-year Expected Shortfall(TVaR))
    - **Natural hazard pool** modelled as the market-wide claims, then the market share is applied
      - Major claims are modelled with
        - Generalized Pareto distribution with threshold= CHF 50m /a=1,25 und b=18,8
        - Max claim for the market-wide event is CHF 500m
        - Frequency: Poisson with lambda = 0.67
    - **Other natural hazards** modelled (business interruption):
      - 20% of the NHP claims for property (fully correlate with the NHP Claims)
      - Max claim for the market-wide event is market share × CHF 1bn
    - **Modelling of cumulated claims due to hail events for comprehensive motor vehicle insurance**
      - Pareto distribution for claim size with alpha=1,85 and threshold= CHF 45m for the market-wide event
      - Poisson distribution for the frequency of claims with lambda=0,9 for the market-wide event

Source: Technical document on the Swiss Solvency Test

Introduction – Importance of NatCat model

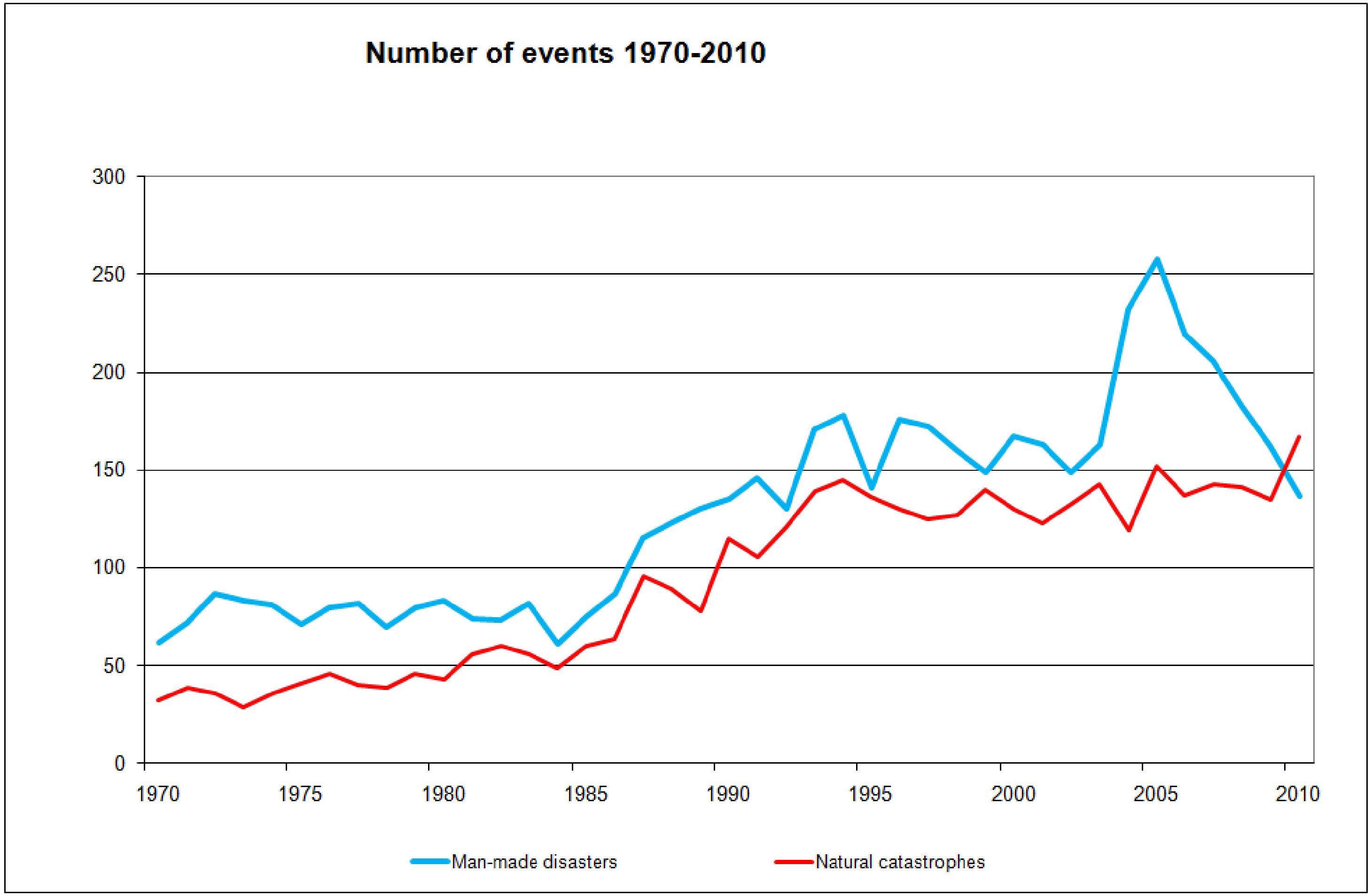
- **Increased risk for Insurance Industry / losses out of catastrophic events**



Source: Sigma1_2011

Introduction – Importance of NatCat model

- **Increased risk for Insurance Industry / number of catastrophic events**



Source: Sigma1_2011

Introduction - Comparison of different NatCat modeling Approaches

- **Physical NatCat models (exposure based, synthetic event catalogue)**
  - High time/space required for computing
  - Scientific view on vulnerability from many perspectives
  - Modeling insights: Black Box Character

- **Stochastic NatCat models (historical experience based)**
  - Only possible having long time observations
  - Usually not sufficient data available (low/no data for earthquake)
  - High transparency

- **Zonal Systems**
  - Distribution of the risks into different vulnerability zones
  - Used in pricing/rating
  - What is the size of PML?

- **Scenario-based approach**
  - QIS5

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History of NatCat vendors models

- Property Insurance
  - Mapping the risk on a wall-hang map

  1800-1960

  Development of GIS Systems

- Natural Hazard Science
  - Understanding nature and impact of natural hazards (measuring hazard intensity)

  1800-seismograph, anemometer

  1970- study about the frequency of NatCat events

- Computer-based models
  - Provide estimates of NatCat losses by overlapping the property at risk with the potential natural hazard sources in the geographical area

  AIR(87), RMS(88), EQE(94)

- 21.09.1989 – Hurricane Hugo, 4 billion insurance loss (South Carolina)
- 17.10.1989 – Earthquake Loma Prieta, 6 billion insurance loss (San Francisco)
- 08.1992 – Hurricane Andrew, 15.5 billion insurance loss (Florida), AIR est. 13 bn. (9 insurance become insolvent) → Need to estimate NatCat risk more precisely

1997 – HAZUS – open source FEMA model to assess EQ Risk in US

2004 – HAZUS-MH – included Wind and Flood

Overview of existing NatCat vendors models

- **Risk Management Solutions (RMS)** was formed in **1988** at **Stanford University**
  - RMS **models risk in over 100 countries**, allowing stakeholders to **analyze the probability of loss in regions with the highest exposure**. Our models are built using detailed data reflecting highly localized variations in hazard, and databases capturing property and human exposures. They are **continually updated with the latest scientific research and data.**

Risk Management Solutions RMS™

- **EQECAT** began in **San Francisco in 1994** as a subsidiary of **EQE International**. In **2001**, EQE International became a part of **ABS Consulting**.
  - Through its modeling software platform, **WORLDCATenterprise™,** EQECAT enables clients **to quantify** and **manage** the potential **financial impact of natural hazards**. WORLDCATenterprise™ includes **181** natural hazard software **models** for **95 countries** spanning six continents. These models are **based** upon innovative applications of the **latest science, engineering expertise, claims and exposure data and advanced mathematics.**

EQECAT AN ABS GROUP COMPANY

Overview of existing NatCat vendors models

- **AIR Worldwide** was founded in **1987 in Boston**
  - AIR Worldwide (AIR) is the scientific leader and most respected provider of risk modelling software and consulting services. AIR founded the catastrophe modelling industry **in 1987** and today models the **risk from natural catastrophes and terrorism in more than 90 countries**. More than **400 insurance, reinsurance, financial, corporate, and government clients rely on AIR** software and services for **catastrophe risk management, insurance-linked securities, detailed site-specific wind and seismic engineering analyses, agricultural risk management, and property replacement-cost valuatio**n. AIR is **a member** of **the Verisk Insurance Solutions group** at Verisk Analytics and is headquartered in Boston with additional offices in North America, Europe, and Asia.

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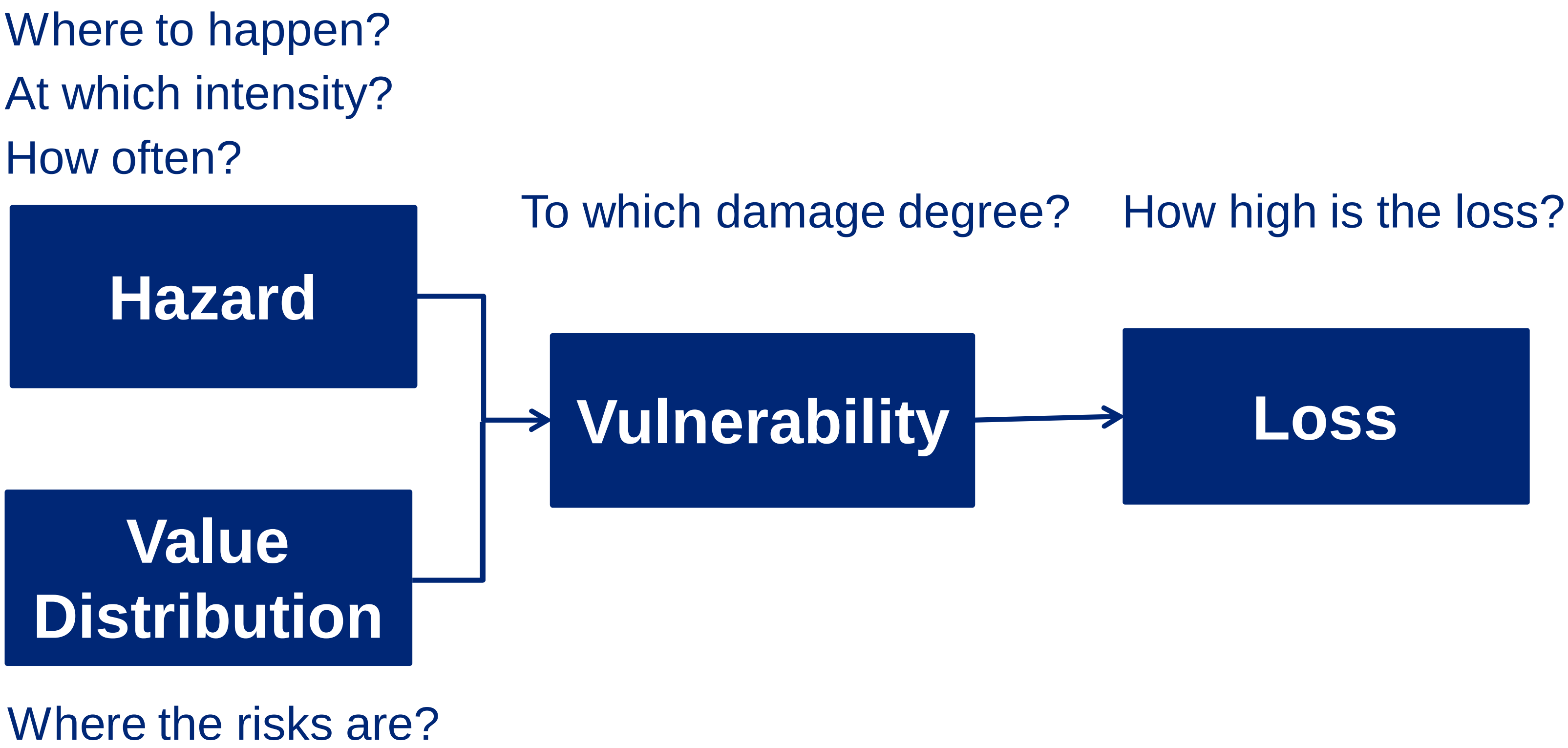
Structure of physical NatCat models

- **The four basic components are:**



Structure of physical NatCat models

- **Hazard module**:

Historical event catalogue

(Lothar, Martin,...)

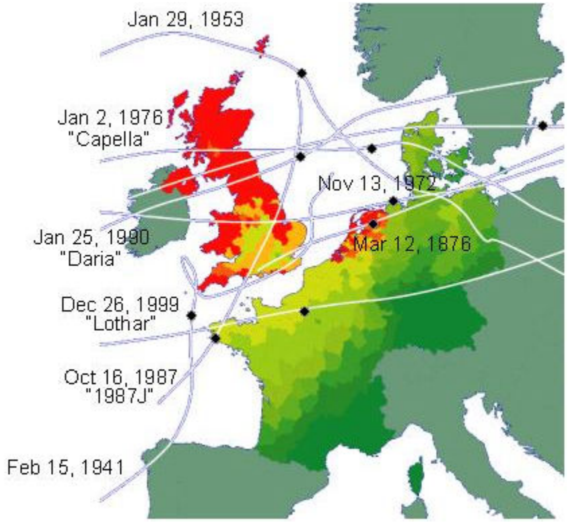
↓

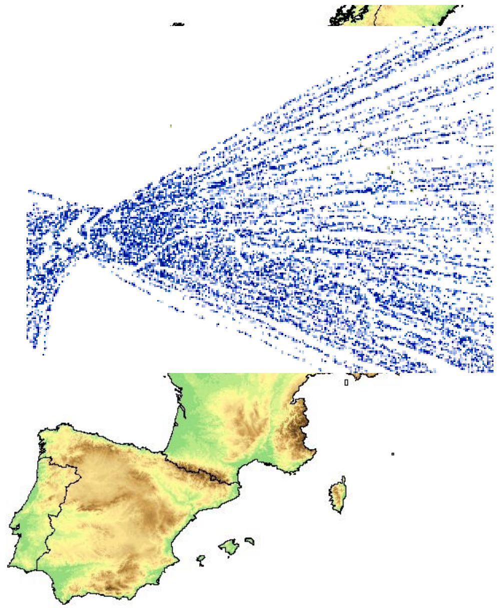
Stochastical sampling

(Winddirection, Wind duration,...)

↓

Complete catalogue of syntetical storm events





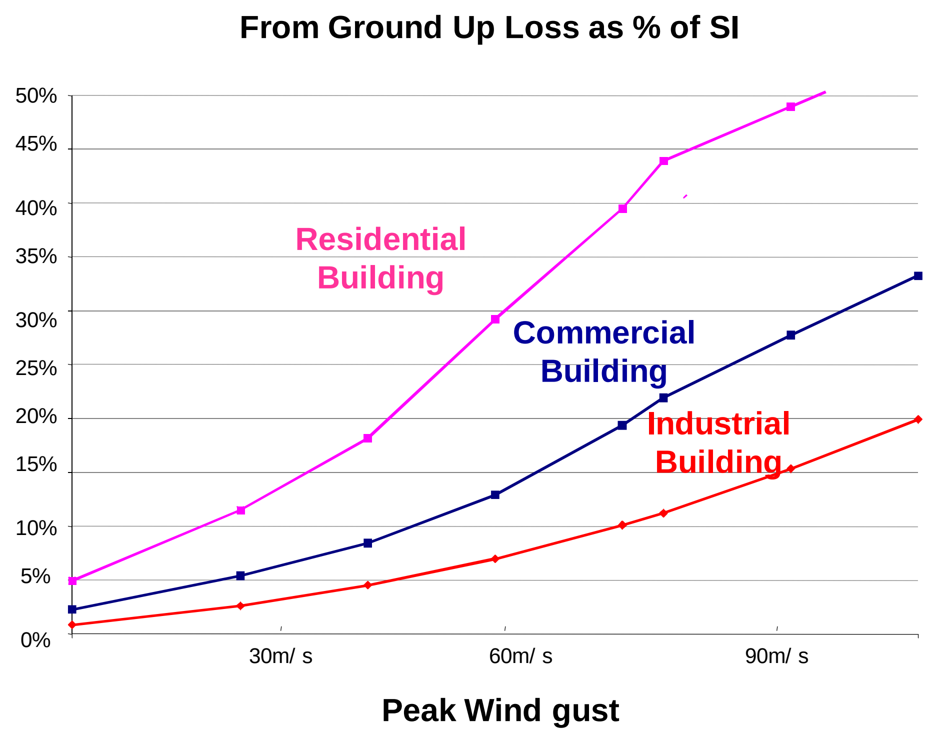
Structure of physical NatCat models

- **Value Distribution module**:
  - Per each location (Cresta Zone, Post Code, Street Level, latitude/longitude)
    - Sum Insured / Number Risks
    - Currency
    - Risk Type (Residential, Commercial, Agricultural, Industrial)
    - Coverage (Building, Contents, BI)
    - Peril covered (Windstorm, Flood, Earthquake, Hail)

LOCATOR	VALUE	CURRENCY	RISKTYPE	COVERAGE	RISKCOUNT	PERIL
DEU-001	49.249.229.746	EUR	R	B	119.884	W
DEU-002	35.432.456.091	EUR	R	B	86.251	W
DEU-003	11.643.429.871	EUR	R	B	28.343	W
DEU-004	56.369.011.698	EUR	R	B	137.215	W
DEU-006	54.569.393.951	EUR	R	B	132.834	W
DEU-007	32.299.802.833	EUR	R	B	78.625	W
DEU-008	28.477.399.949	EUR	R	B	69.320	W
DEU-009	35.815.302.113	EUR	R	B	87.183	W
DEU-010	60.616.808.023	EUR	R	B	147.555	W
DEU-012	189.621.997.488	EUR	R	B	461.583	W
DEU-013	159.821.737.633	EUR	R	B	389.042	W
DEU-014	111.335.190.915	EUR	R	B	271.015	W
DEU-015	37.890.315.473	EUR	R	B	92.234	W
DEU-016	36.405.627.432	EUR	R	B	88.620	W
DEU-017	43.131.021.955	EUR	R	B	104.991	W
DEU-018	21.301.685.219	EUR	R	B	51.853	W
DEU-019	26.078.197.255	EUR	R	B	63.480	W
DEU-020	16.281.823.046	EUR	R	B	39.634	W
DEU-021	181.363.083.176	EUR	R	B	441.479	W
DEU-022	131.953.304.829	EUR	R	B	321.204	W

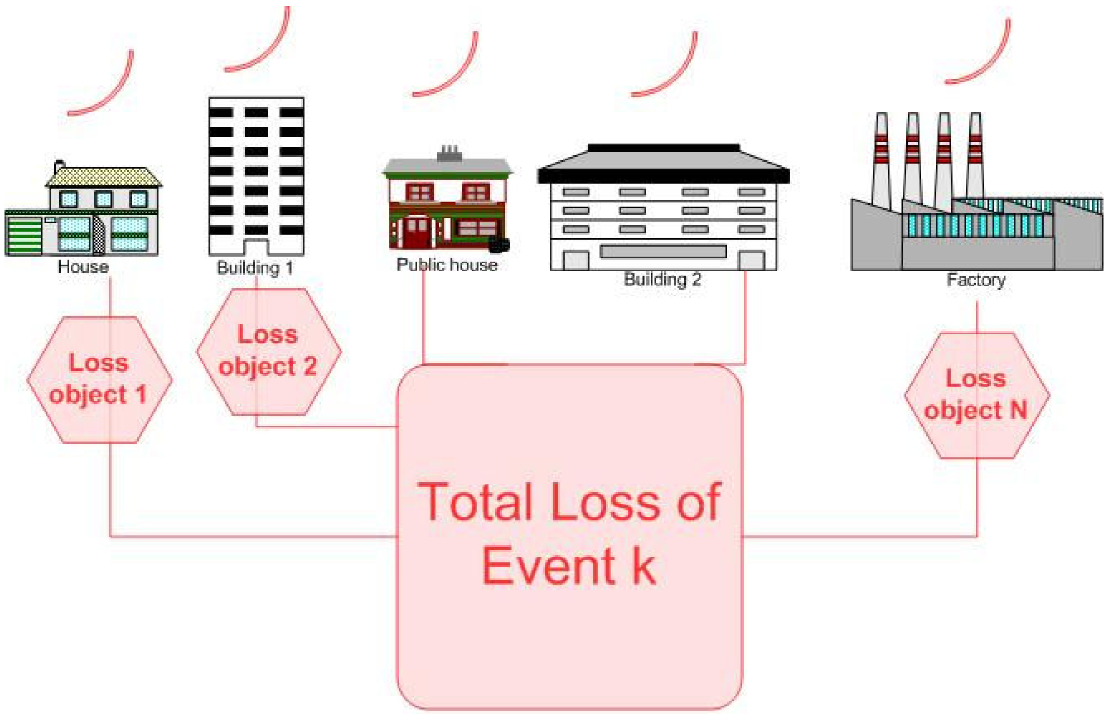
Structure of physical NatCat models

- **Vulnerability module:**
  - The vulnerability function converts the intensity of the event (magnitude, water height, wind speed) into loss amounts for each object within a calculation unit
  - Different vulnerability functions exist for different occupancy type: Residential, Commercial, Industrial and Agricultural
  - Further refinement can be done due to e.g. the age and construction type of the building (secondary modifiers)
  - Calibrated using historical loss experience, engineering analysis of building types and expert judgement



Structure of physical NatCat models

- **Loss module:**
  - The damage ratio distribution for a specific event is then multiplied by the building replacement value to obtain the loss per event
  - Here insurance policy conditions like limits, deductibles and reinsurance terms are incorporated
  - The combined loss distribution of all buildings is computed using convolution process
  - Synthetic loss history of x.000 Years is simulated



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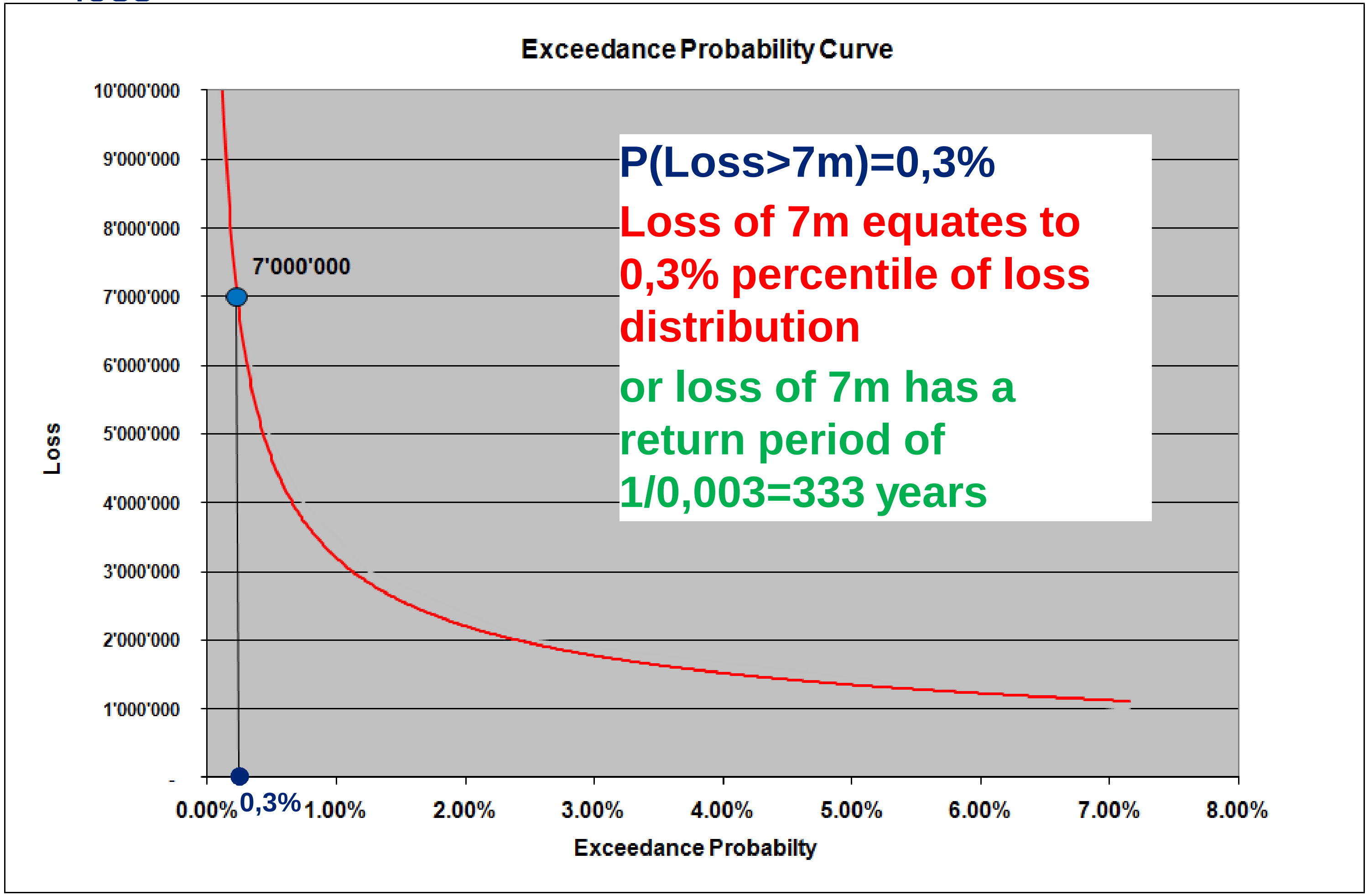
Output and Terminology of physical NatCat models

- **Exceedance probability curves (EP Curve):**
  - Annual Exceedance probability curve (AEP) – aggregated annual loss
  - Occurence Exceedance probability curve (OEP) – max occurrence loss



Output and Terminology of physical NatCat models

Output: Event by event results - EQECAT

LOCATION	SCENARIO	TITLE	FREQUENCY	HAZARD	Total Insured Value	Mean Loss	Extreme Loss	Standard Deviation of Loss
CO	2797	point, mag. 7.25 4.75, -74.00	0.00003651	Earthquake	3,658,642,106,949	2,344,540,743,385	2,875,208,611,088	414,082,224,919
CO	2796	point, mag. 7.25 4.50, -74.00	0.00005488	Earthquake	3,658,642,106,949	2,287,517,002,843	2,806,177,027,351	404,712,458,419
CO	2773	point, mag. 7.25 4.75, -74.25	0.00003472	Earthquake	3,658,642,106,949	1,929,387,861,125	2,372,683,388,678	345,905,251,284
CO	2772	point, mag. 7.25 4.50, -74.25	0.00004931	Earthquake	3,658,642,106,949	1,891,509,208,098	2,326,839,596,887	339,690,023,046
CO	2408	point, mag. 7.00 4.75, -74.00	0.00001953	Earthquake	3,658,642,106,949	1,678,627,012,064	2,069,220,910,502	304,781,969,813
CO	2407	point, mag. 7.00 4.50, -74.00	0.00001958	Earthquake	3,658,642,106,949	1,621,603,271,522	2,000,223,428,200	295,438,814,174
CO	2391	point, mag. 7.00 4.75, -74.25	0.00002310	Earthquake	3,658,642,106,949	1,263,474,129,826	1,567,028,115,443	236,864,410,036
CO	2390	point, mag. 7.00 4.50, -74.25	0.00002319	Earthquake	3,658,642,106,949	1,225,595,476,823	1,521,227,599,452	230,682,953,821
CO	2823	point, mag. 7.25 4.75, -73.75	0.00003459	Earthquake	3,658,642,106,949	1,133,276,934,507	1,409,978,582,652	215,911,447,293
CO	2822	point, mag. 7.25 4.50, -73.75	0.00004896	Earthquake	3,658,642,106,949	1,125,276,980,180	1,400,373,667,514	214,659,092,162

3,006 events in total.

Output and Terminology of physical NatCat models

Output: Event by Event results - RMS

Event ID	Gross Loss	Std. Deviation (Independent)	Std Deviation (Correlated)	Exposure Value	Annual Rate
97764	1,034,948,739,027	66,722,063,262	76,024,369,742	1,847,237,388,468	0.0029700
97765	768,981,532,454	60,572,602,523	69,447,871,541	1,543,574,199,604	0.0009476
97766	768,981,532,454	60,572,602,523	69,447,871,541	1,543,574,199,604	0.0009476
97705	621,255,289,459	54,517,087,839	62,641,066,743	1,898,735,000,321	0.0018289
97767	525,645,970,956	49,645,193,559	57,135,053,483	1,778,456,278,131	0.0007749
97768	525,645,970,956	49,645,193,559	57,135,053,483	1,778,456,278,131	0.0007749
97769	525,645,970,956	49,645,193,559	57,135,053,483	1,778,456,278,131	0.0007749
97770	525,645,970,956	49,645,193,559	57,135,053,483	1,778,456,278,131	0.0007749
97706	406,126,273,404	44,712,954,333	51,572,751,650	3,586,392,898,026	0.0011682
100967	343,530,261,695	40,876,542,310	47,147,574,661	2,953,479,168,380	0.0007870

1,650 EQ events in RMS model event set

Output and Terminology of physical NatCat models

Output: Event by Event results - AIR

* Year	Event C	Total.GU	Total.GR	Total.NT	Event Desc
1	1	88548925	88548925	88548925	Wind Event:
1	5	6993202	6993202	6993202	Wind Event:
1	4	2405710	2405710	2405710	Wind Event:
6	14	20371094	20371094	20371094	Wind Event:
6	16	466315	466315	466315	Wind Event:
7	18	1129235	1129235	1129235	Wind Event:
8	26	124815193	124815193	124815193	Wind Event:
9	30	113495007	113495007	113495007	Wind Event:
9	28	17941380	17941380	17941380	Wind Event:
10	33	868361	868361	868361	Wind Event:
10	32	368008	368008	368008	Wind Event:
11	35	590664791	590664791	590664791	Wind Event:
12	37	179151	179151	179151	Wind Event:
12	38	103798	103798	103798	Wind Event:
13	43	865497771	865497771	865497771	Wind Event:
14	49	230316096	230316096	230316096	Wind Event:
14	47	23463630	23463630	23463630	Wind Event:
15	53	1091451533	1091451533	1091451533	Wind Event:

Agenda

- **Introduction**
  - Definition of NatCat events
  - Use of the NatCat models
  - Insights into regulatory view
  - Importance of the NatCat models
  - Comparison of different NatCat Approaches

- **Physical NatCat models**
  - History of NatCat vendor models
  - Overview of existing NatCat vendors models
  - Structure of the models or Approach of physical NatCat models
  - Output and Terminology of physical NatCat models
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  - Advantages/ disadvantages in using the Physical NatCat models

- **Future prospects**

- **Literature**

How to incorporate the results into the internal solvency models

Event Loss Set:

- Each row of the ELS represents a collective risk model with the parameter λ_i for the Frequency distribution of the event *i*, *i=1,..,n* and the parameter α_i und β_i for the Size distribution of the event *i* , *i*=1,..,*n*. (*n* – is the number of rows or the number of events in the ELS)
- Under the assumption of Poisson distribution for the Frequency N_i for the Event *i*, is the total loss distribution for the Event *i* , *i*=1,..,*n :*

$$S_i = X_{i,1} + X_{i,2} + \cdots + X_{i,N_i} \sim P_i = \sum_{k=0}^{\infty} R_i\{k\} Q_i^{*k}$$

– Loss Number N_i is Poisson distributed with the Parameter λ_i

– Loss Size $X_{i,1}, X_{i,2}, \dots, X_{i,N_i} \sim Q_i$ is Beta distributed with the Parameters α_i und β_i

How to incorporate the results into the internal solvency models

Theorem:

- Are $\mathrm{N}_1, \dots, \mathrm{N}_\mathrm{n}$ stochastically Independent Poisson distributed random variables with the parameters $\lambda_1, \dots, \lambda_\mathrm{n} > 0$ and $\mathrm{X}_{\mathrm{i,j}}$ with $0 \le i \le n, j \in Z_+$ positive independent also of N_i as Q_i distributed random variables.
- Then is the distribution of

$$\mathrm{S} := \sum_{i=1}^{n} \sum_{\mathrm{j}=1}^{\mathrm{N}_\mathrm{i}} \mathrm{X}_{\mathrm{i,j}} \sim \mathrm{P} := \mathrm{P}_1 * \cdots * \mathrm{P}_\mathrm{n} = \mathrm{PSV}(\lambda, \mathrm{Q})$$

- With $\lambda_1 + \cdots \lambda_\mathrm{n} = \lambda$ and $\mathrm{Q} = \sum_{\mathrm{i}=1}^{\mathrm{n}} \frac{\lambda_1}{\lambda} \mathrm{Q}_\mathrm{i}$

- *n* – is the number of rows in the ELS or number of Event in the ELS

How to incorporate the results into the internal solvency models

With all these assumption can AEP and OEP curve be directly calculated using the ELS

$$M := \max\left\{X_{i,j}, 1 \le i \le n, 1 \le j \le N_i\right\}, S := \sum_{i=1}^{n} \sum_{j=1}^{N_i} X_{i,j}$$

- OEP Curve – can be calculated directly from the ELS

$$P(M > x) = 1 - \exp\left\{-\lambda[1 - Q(x)]\right\} \qquad \text{(OEP-Kurve)}$$

- AEP Curve – Simulation, Panjer Algorithms, FFT

$$P(S > x) = 1 - e^{-\lambda} - e^{-\lambda} \sum_{i=1}^{n} \frac{\lambda^i}{i!} Q^{*i}(x) \quad \text{(AEP-Kurve)}$$

- With $\lambda_1 + \cdots \lambda_n = \lambda$ and $Q = \sum_{i=1}^{n} \frac{\lambda_1}{\lambda} Q_i$
  - n – ist die Anzahl der Zeilen oder Anzahl der Ereignisse im ELS

How to incorporate the results into the internal solvency models

1. Option

Event Loss Set → e.g. ReMetrica / Igloo

2. Option

Event Loss Set → In –house internal model / Excel (VBA)

3. Option

Frequency / Severity Distribution → In-house internal model / Excel (VBA)

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Advantages/Disadvantages in using the physical NatCat models

- **Advantages:**
  - The results can be easily incorporated into the internal solvency models using Event Loss Table Outputs (ELT)
  - The results are accepted and required by the rating agency

- **Disadvantages:**
  - Model changes
  - Black Box Character
  - Model understanding
  - Model uncertainty

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Future Prospects

- „Open source“ models, jointly developed by scientists, engineers and industry sectors for each single risk

Literatur

P.Grosssi, H. Kunreuther, „Catastrophy Modelling: A new approach to managing risk“

W. Dong, „Modern Portfolio Theory with Application to Catastrophe Insurance“ (RMS)

M. Khater, D. Kuzak, „Natural Catastrophe Loss Modelling“ (EQECAT)

D. Pfeifer, „Grundzüge der Versicherungsmathematik“

M. Wilkinson, „Estimating probable maximum loss with order statistics“ , PCAS 59

D. Pfeifer, „How to calculate confidence bounds for PML estimates based on simulation“

C. Hipp, D. Pfeifer, „Risikotheoretische Konzepte unter Maple und Excel

Swiss Re, „Naturkatastrophen und Rückversicherung“

Thomas Mack: Schadenversicherungsmathematik, 2. Auflage, VVW Karlsruhe, 2002

Stuart A. Klugman, Harry H. Panjer, Gordon E. Wilmot: Loss Models. From Data to Decisions. Wiley, N.Y. 1998.

Thank you very much for your Attention