Halb-Mack Stochastische Reservierung

Bahnhofskolloquium

Dezember 2013

A Loss Development

US NAIC Schedule P Data: medical malpractice

2004	217’239
2005	222’707
2006	235’717
2007	275’923
2008	267’007
2009	276’235
2010	252’449
2011	209’222
2012	107’474
2013	20’361

A Loss Development

	1	2	3	4	5	6	7	8	9	10
2004										217’239
2005									222’707	
2006								235’717		
2007							275’923			
2008						267’007				
2009					276’235					
2010				252’449						
2011			209’222							
2012		107’474								
2013	20’361									

A Loss Development Triangle

	1	2	3	4	5	6	7	8	9	10
2004		47'258	95'054	131'616	165'117	181'937	198'395	206'068	213'374	217'239
2005	9'433	51'855	98'976	149'169	175'588	197'590	211'242	217'986	222'707	
2006	11'996	54'742	118'964	163'695	190'391	213'972	225'199	235'717		
2007	9'517	73'420	146'347	199'262	244'987	260'333	275'923			
2008	12'479	78'212	157'400	209'959	244'018	267'007				
2009	18'229	90'710	166'325	227'891	276'235					
2010	14'952	94'303	186'577	252'449						
2011	17'995	110'181	209'222							
2012	20'390	107'474								
2013	20'361									

accident years

calendar years

development years

A Loss Development Triangle

	1	2	3	4	5	6	7	8	9	10
2004	5'934	47'258	95'054	131'616	165'117	181'937	198'395	206'068	213'374	217'239
2005	9'433	51'855	98'976	149'169	175'588	197'590	211'242	217'986	222'707	
2006	11'996	54'742	118'964	163'695	190'391	213'972	225'199	235'717		
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2011	17'995	110'181	209'222							
2012	20'390	107'474								
2013	20'361									

A Loss Development Square

	1	2	3	4	5	6	7	8	9	10
2004	5'934	47'258	95'054	131'616	165'117	181'937	198'395	206'068	213'374	217'239
2005	9'433	51'855	98'976	149'169	175'588	197'590	211'242	217'986	222'707	
2006	11'996	54'742	118'964	163'695	190'391	213'972	225'199	235'717		
2007	9'517	73'420	146'347	199'262	244'987	260'333	275'923			
2008	12'479	78'212	157'400	209'959	244'018	267'007				
2009	18'229	90'710	166'325	227'891	276'235					
2010	14'952	94'303	186'577	252'449						
2011	17'995	110'181	209'222							
2012	20'390	107'474								
2013	20'361									

calendar year 2022

A Loss Development Square

	1	2	3	4	5	6	7	8	9	10
2004	5’934	47’258	95’054	131’616	165’117	181’937	198’395	206’068	213’374	217’239
2005	9’433	51’855	98’976	149’169	175’588	197’590	211’242	217’986	222’707	224’455
2006	11’996	54’742	118’964	163’695	190’391	213’972	225’199	235’717	241’665	244’504
2007	9’517	73’420	146’347	199’262	244’987	260’333	275’923	285’629	291’979	294’358
2008	12’479	78’212	157’400	209’959	244’018	267’007	283’515	291’797	300’773	305’375
2009	18’229	90’710	166’325	227’891	276’235	306’495	320’739	334’502	340’161	346’427
2010	14’952	94’303	186’577	252’449	297’167	326’180	343’125	352’447	363’673	369’122
2011	17’995	110’181	209’222	274’772	322’117	355’709	369’967	383’721	392’424	399’426
2012	20’390	107’474	203’137	268’721	322’309	355’155	373’739	383’606	390’761	394’640
2013	20’361	112’636	229’414	312’250	362’737	395’955	414’758	430’788	437’228	447’198

ultimates

US NAIC Schedule P Data

- http://www.casact.org/research/index.cfm?fa=loss_reserves_data
- Many carriers
- 6 lines of business
  1. personal auto
  2. commercial auto
  3. medical malpractice
  4. workers compensation
  5. general liability
  6. product liability

Agenda

- Mack Chain Ladder Procedure
- Half-Mack Procedure
- Half-Mack Sampling

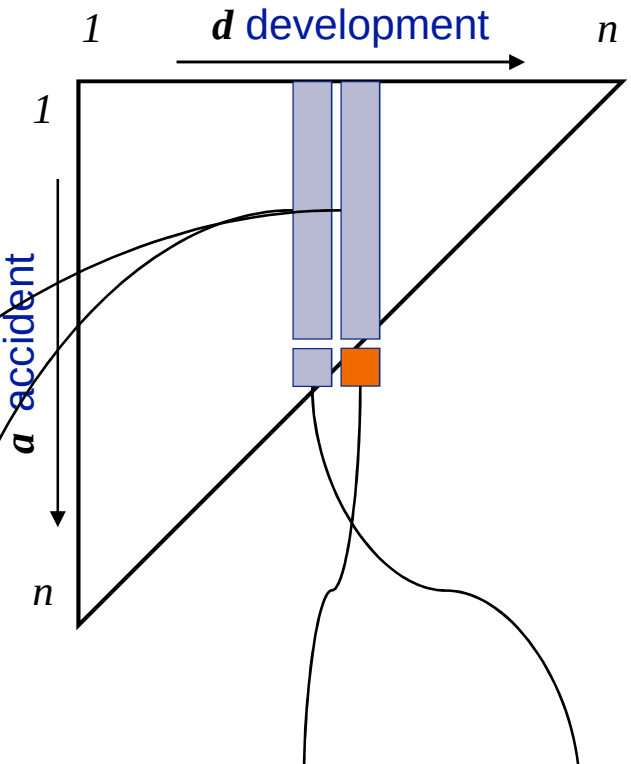
Chain Ladder Procedure

Notations:

a = accident year

d = development year

$C_{a,d}$ = cumulative losses

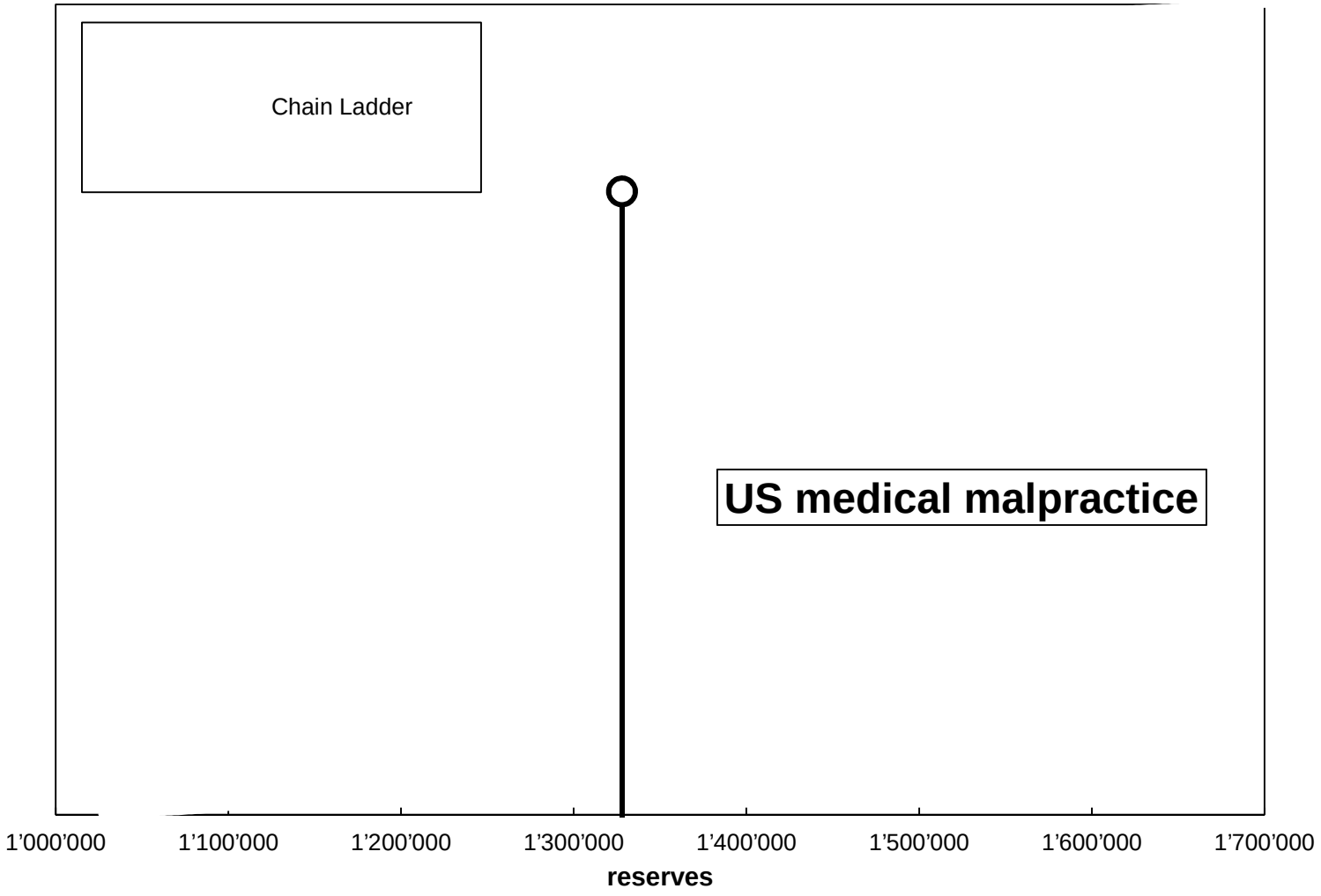


Algorithm:

$$\mathsf{a}_{d-1} = \sum_{a=1}^{n-d+1} C_{a,d} \Big/ \sum_{a=1}^{n-d+1} C_{a,d-1}$$

$$C_{a,d} = \mathsf{a}_{d-1} \cdot C_{a,d-1}$$

Chain Ladder Reserves



Mack Chain Ladder Procedure

Hypothesis:

$C_{a,d}$ independent for each accident year a

$$E\left[C_{a,d}\middle|C_{a,1}\cdots C_{a,d-1}\right] = \alpha_{d-1}\cdot C_{a,d-1}$$

$$V\left[C_{a,d}\middle|C_{a,1}\cdots C_{a,d-1}\right] = \pm_{d-1}\cdot C_{a,d-1}$$

- Variance proportional to the mean
- Same development pattern for all accident years
- Future depends only on the last diagonal
- Independent accident years

Mack Chain Ladder Procedure

PRS EXPERTS FOR EXPERTS

Hypothesis:

$C_{a,d}$ independent for each accident year a

$$E\left[C_{a,d}\middle|C_{a,1}\cdots C_{a,d-1}\right] = \mathrm{a}_{d-1}\cdot C_{a,d-1}$$

$$V\left[C_{a,d}\middle|C_{a,1}\cdots C_{a,d-1}\right] = \pm_{d-1}\cdot C_{a,d-1}$$

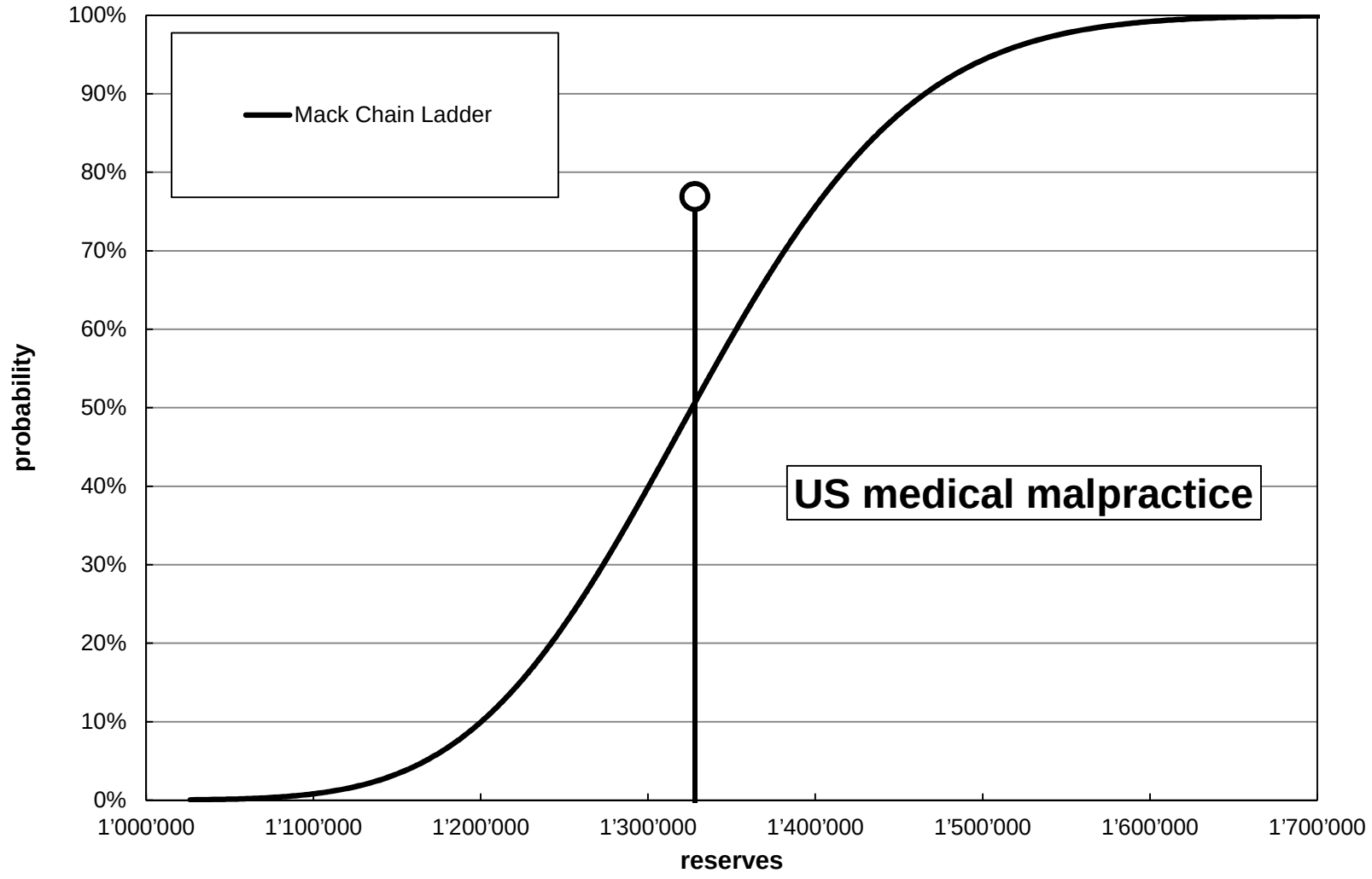
Consequences:

$$\begin{cases} \mathrm{a}_{d-1} = \sum_{a=1}^{n-d+1} C_{a,d} \Big/ \sum_{s=1}^{n-d+1} C_{a,d-1} & \rightarrow \quad E[R] \\ \pm_{d-1} = \dfrac{1}{n-d}\sum_{a=1}^{n-d+1} C_{a,d-1}\left(\dfrac{C_{a,d}}{C_{a,d-1}} - \mathrm{a}_{d-1}\right)^2 & \rightarrow \quad V[R] \end{cases}$$

Application:

- Assume an underlying distribution of the reserves
- Fit with the first 2 moments

Mack Chain Ladder Reserves



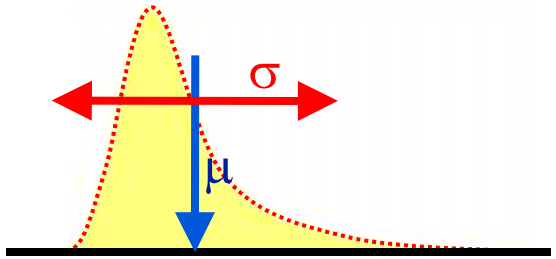
Mack Chain Ladder Procedure

☺

- Systematic & statistical errors
- Non-parametric

☹

- No smoothing/adjustment of the development factors
- No tail factors
- Only 2 moments



Mack Chain Ladder Procedure

PRS EXPERTS FOR EXPERTS

Hypothesis:

$C_{s,d}$ independent for each accident year s

$$E\left[C_{a,d}\middle|C_{a,1}\cdots C_{a,d-1}\right] = \mathsf{a}_{d-1}\cdot C_{a,d-1}$$

$$V\left[C_{a,d}\middle|C_{s,1}\cdots C_{a,d-1}\right] = \pm_{d-1}\cdot C_{a,d-1}$$

Consequences:

$$\begin{cases} \mathsf{a}_{d-1} = \sum_{a=1}^{n-d+1} C_{a,d} \Big/ \sum_{a=1}^{n-d+1} C_{a,d-1} & \rightarrow \quad E[R] \quad E\left[a2u_d\right] \\ \pm_{d-1} = \dfrac{1}{n-d}\sum_{a=1}^{n-d+1} C_{a,d-1}\left(\dfrac{C_{a,d}}{C_{a,d-1}} - \mathsf{a}_{d-1}\right)^2 & \rightarrow \quad V[R] \quad V\left[a2u_d\right] \end{cases}$$

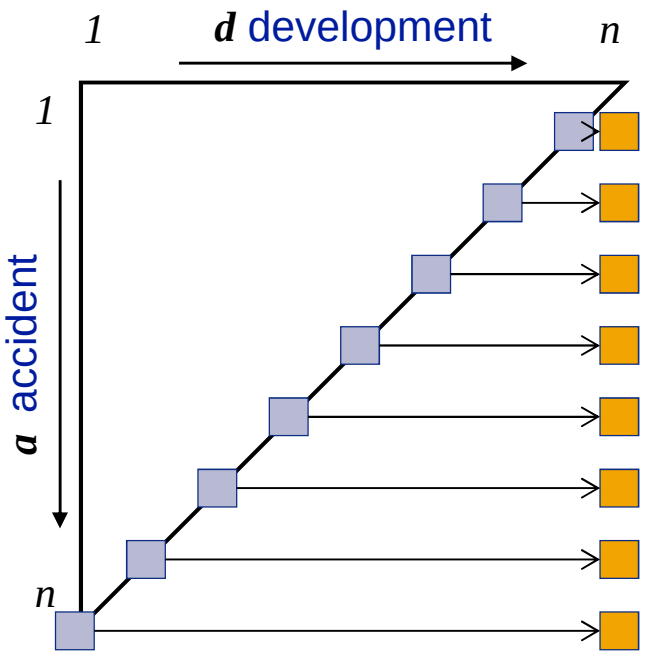
Application:

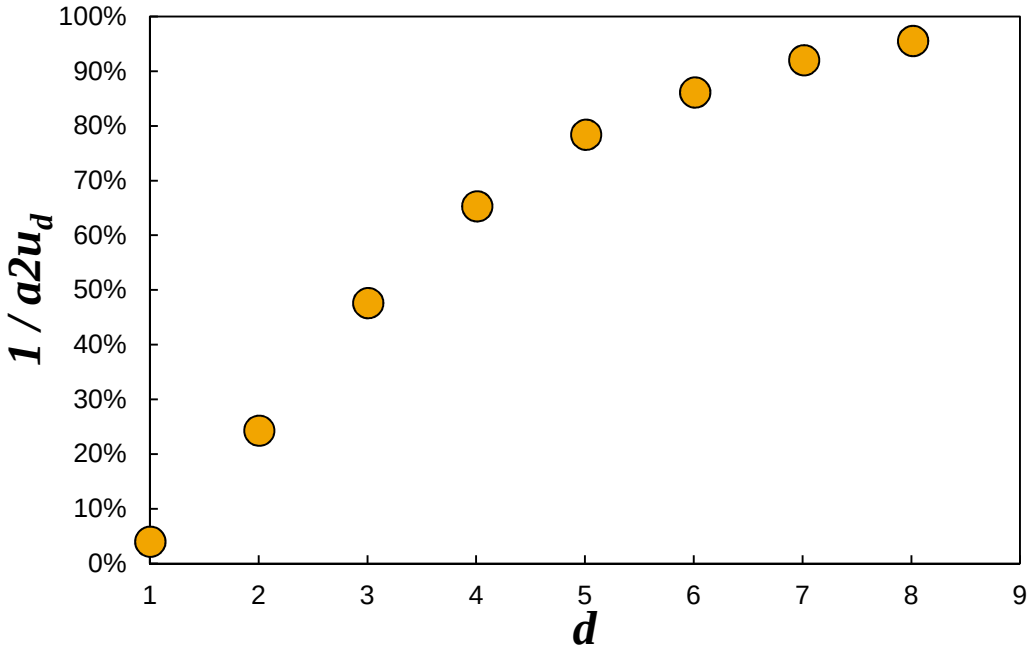
- Assume an underlying distribution of the reserves
- Fit with the first 2 moments
- Take over the first 2 moments only of the development factors
- Fit a model to the development factors

Agenda

- Mack Chain Ladder Procedure
- Half-Mack Procedure
- Half-Mack Sampling

Loss Development Pattern

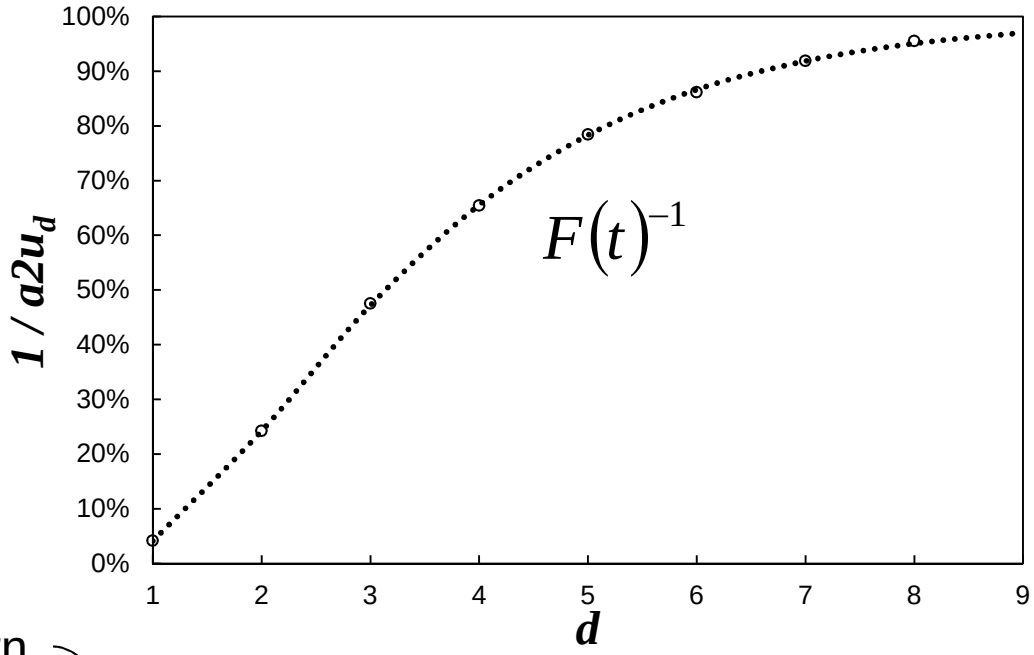




Loss Development Pattern

Reserving actuaries tricks:

- Adjustments to experience
- Smoothing
- Fit to a model



loss development pattern

loss development factor

$$a2u_d \rightarrow F(t) = \left[1 - \exp\left(-\frac{t-2}{a}\right)\right]^{-\tilde{}}$$

location

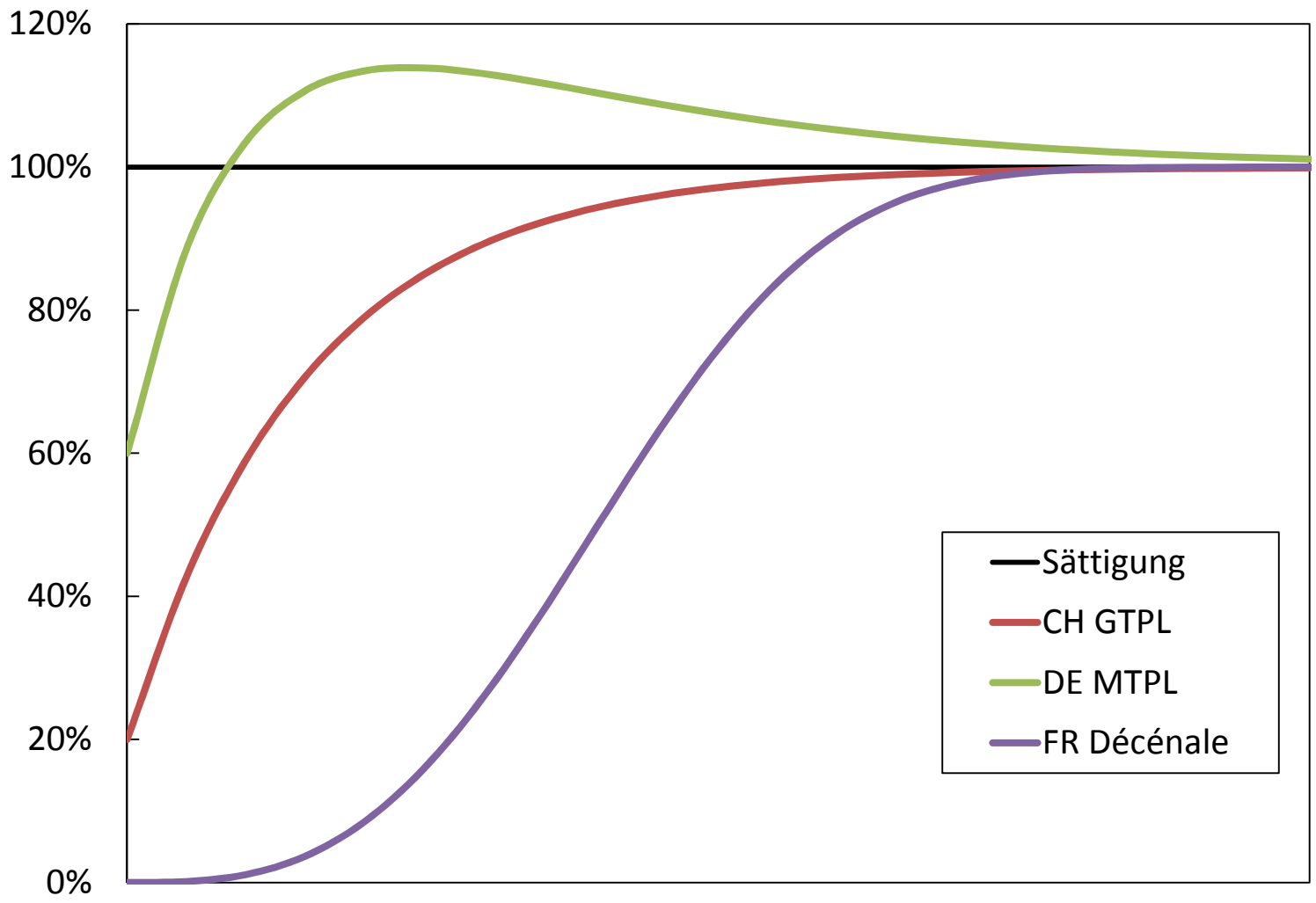
shape

development year

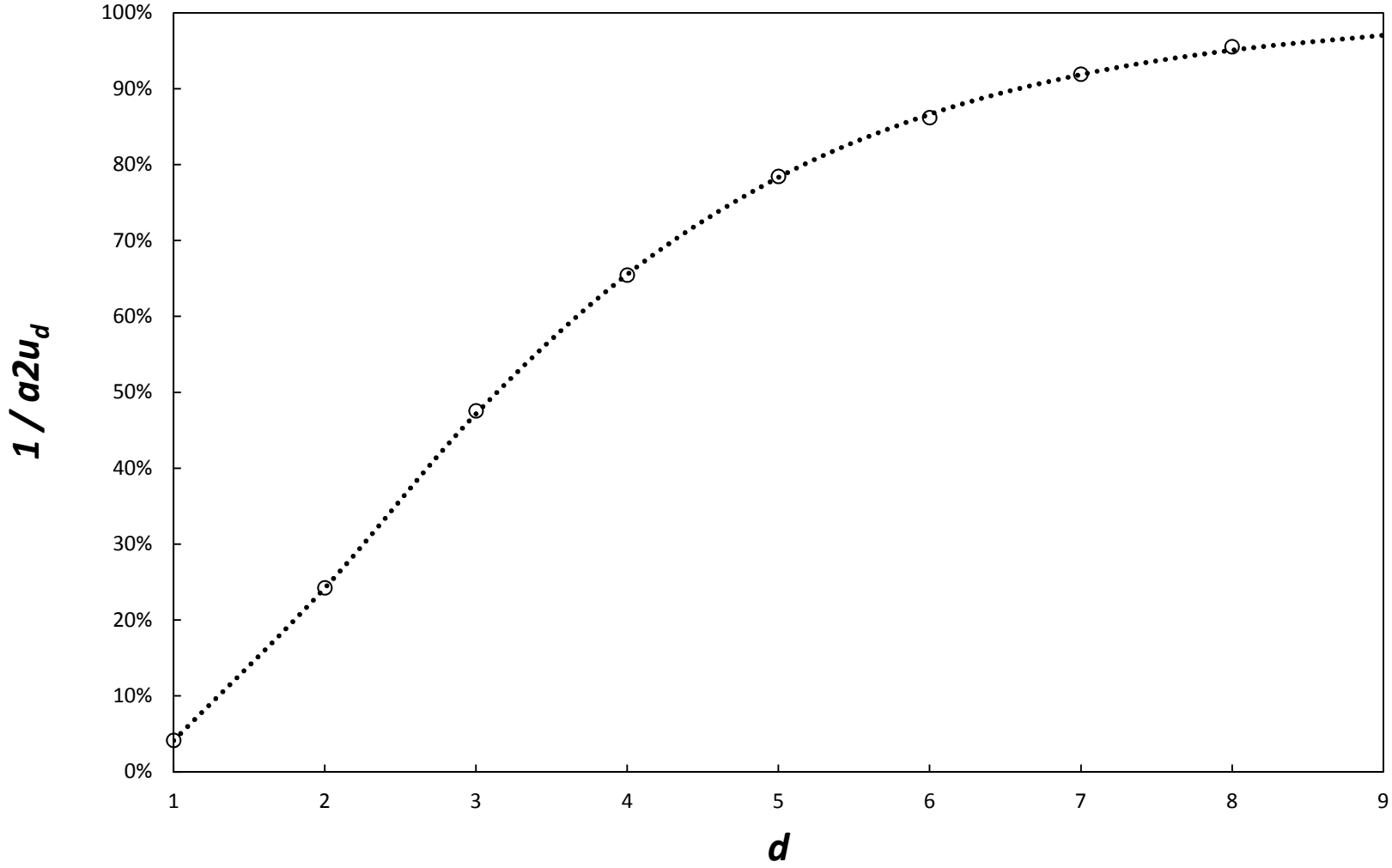
continuous time

scale

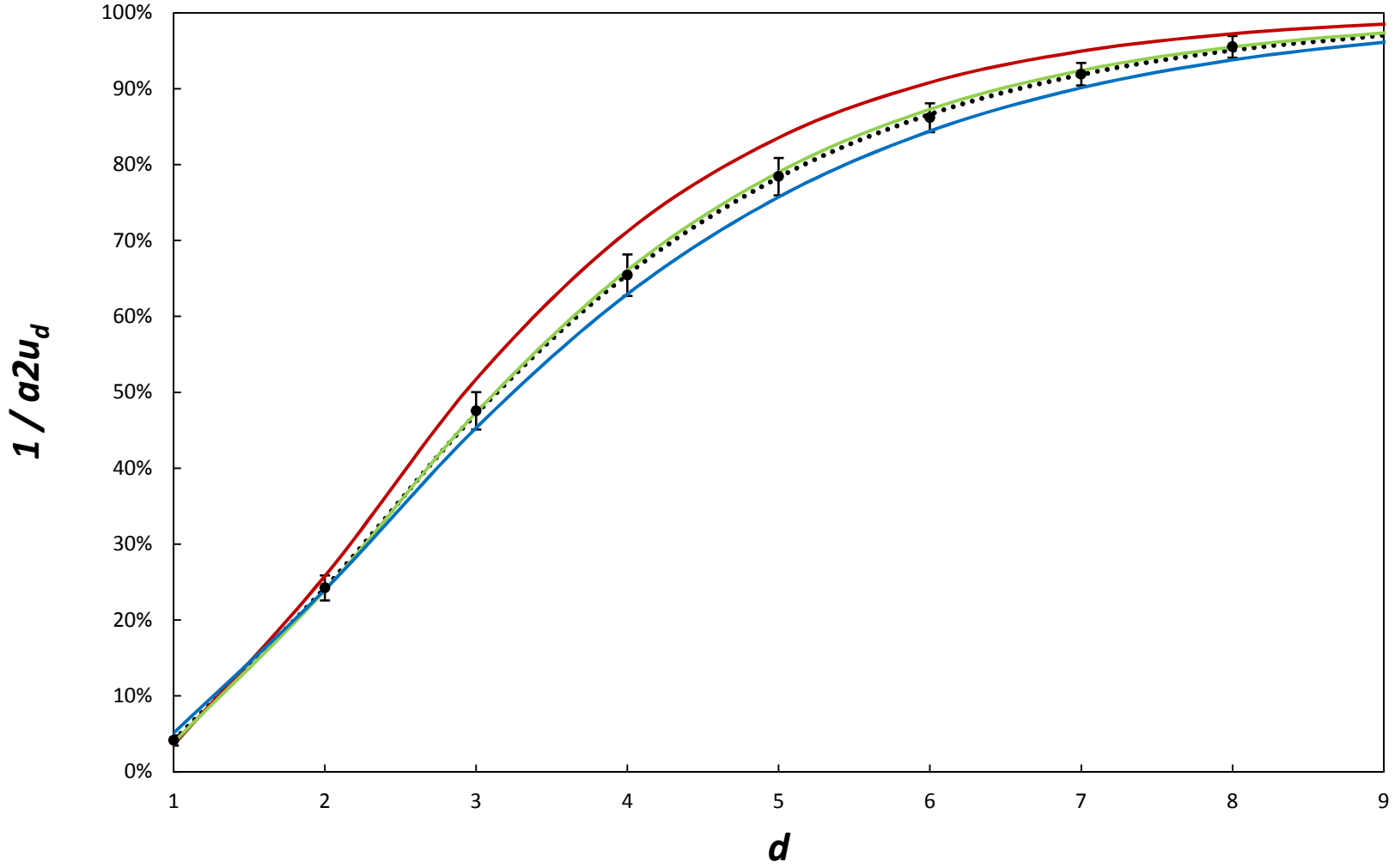
Typical Loss Development Patterns



Loss Development Patterns



Half-Mack Loss Development Patterns



Half-Mack Loss Development Patterns

Fit with χ^2 statistic

- Best estimate minimizes χ^2 function

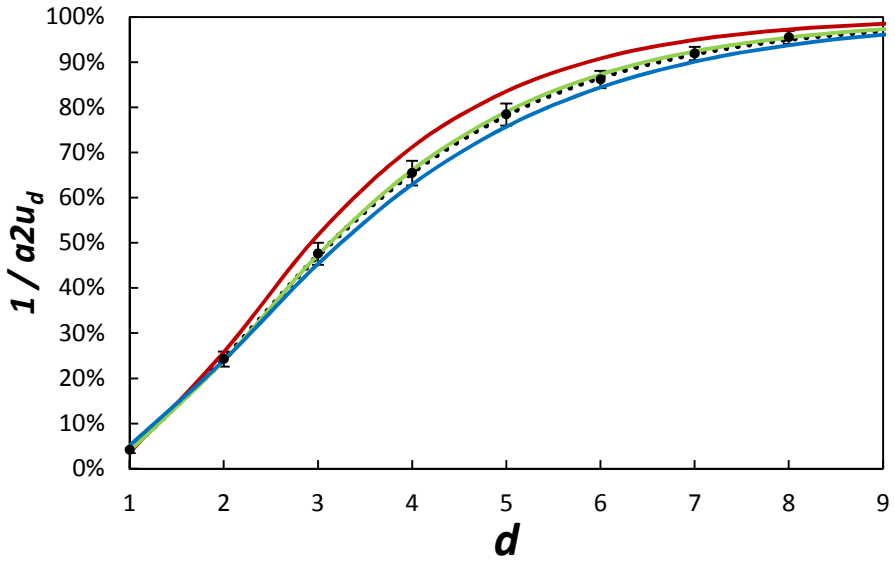
CL estimation

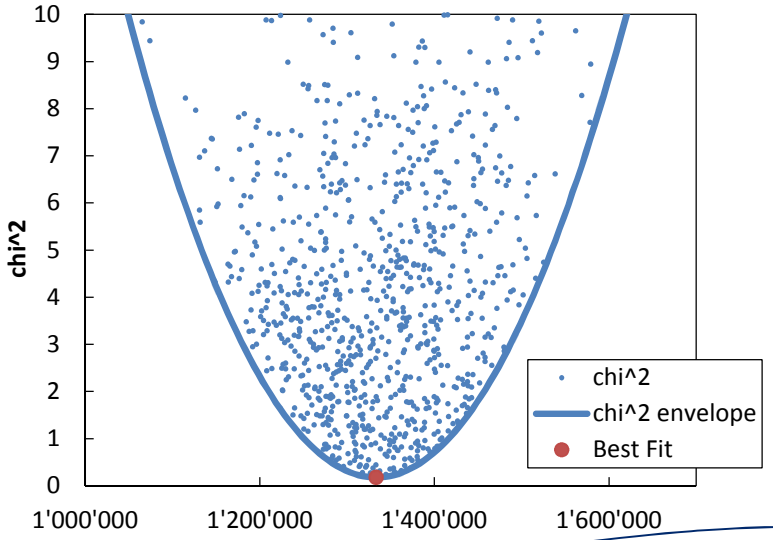
model

$$\chi^2 = \sum_{d=1}^{T} \left(\frac{a2u_d - F(d)}{\Delta a2u_d} \right)^2$$

MCL error

- Goodness of fit $\frac{\chi^2_{min}}{dof}$



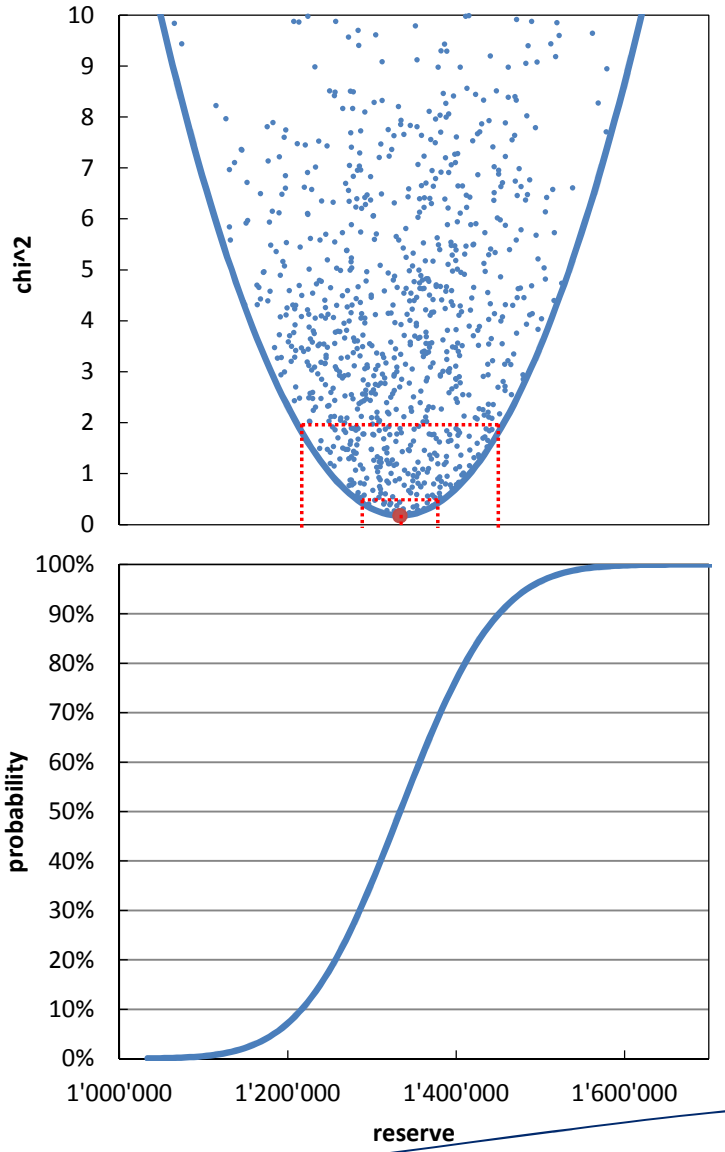


Half-Mack Reserves Distribution

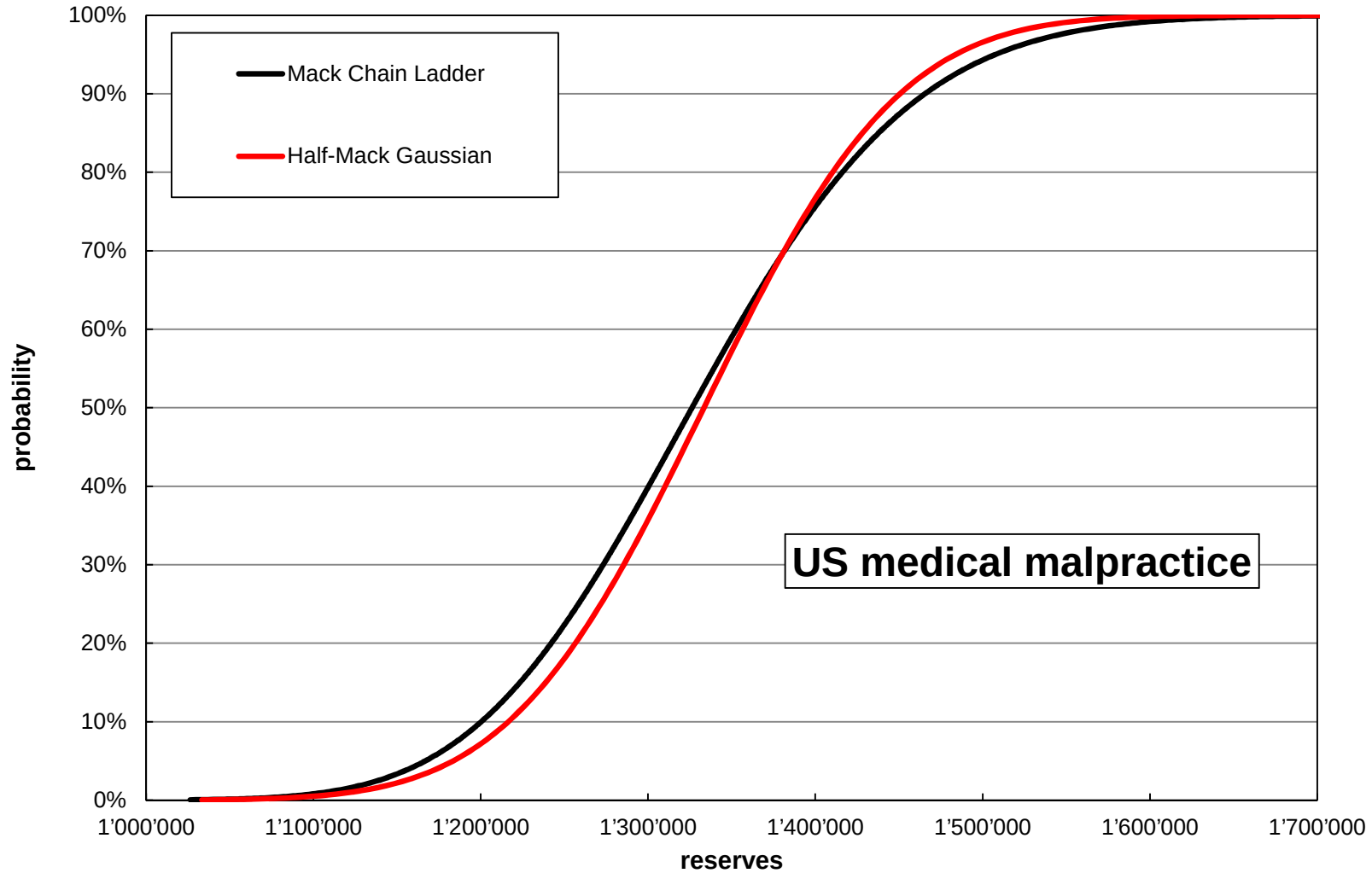
- best fit: $¡^2 = ¡^2_{\min}$

- $¡^2$ confidence intervals
  - $1\pm$ interval: $¡^2 \le ¡^2_{\min} + 1$
  - p intervals: $¡^2 \le ¡^2_{\min} + ¡^2(p)$

→ reserves distribution



Mack Chain Ladder Reserves



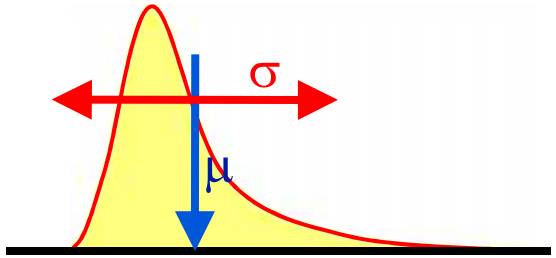
The Mack Chain Ladder Procedure

☺

- Systematic & statistical errors
- Non-parametric
- Natural smoothing of the development factors
- Full distribution of reserves
- Automatic tail factors
- Accounts for market experience

☹

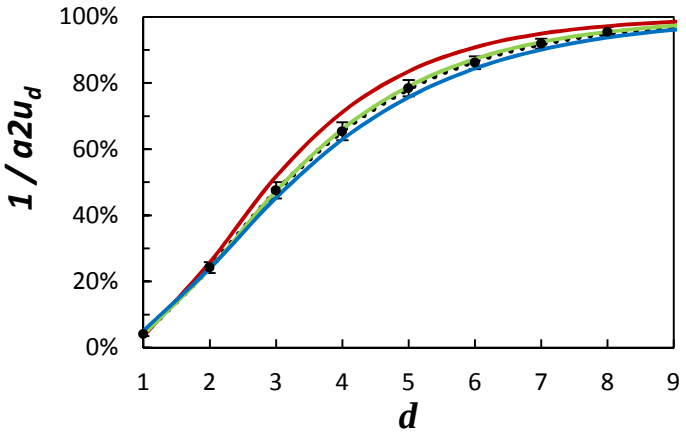
- No smoothing/adjustment of the development factors
- Only 2 moments
- No tail factors



The Half-Mack Procedure

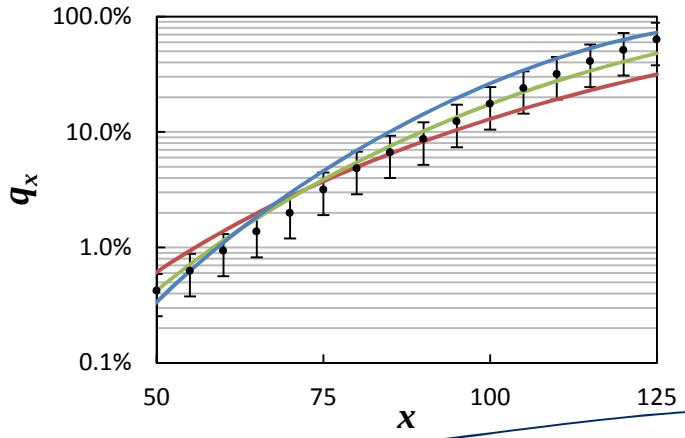
Reserving Risk

- Estimate $a2u_d$ with errors a$a2u_d$
- Fit $a2u_d$ with $F(d)$
- $F(d)$ determines reserves
- $F(d)$ confidence intervals determine reserves distribution



Parameter Risk in General

- Estimate q_x with errors aq_x
- Fit q_x with $G(x)$
- $G(x)$ determines EV
- $G(x)$ confidence intervals determine EV distribution



Agenda

- Mack Chain Ladder Procedure
- Half-Mack Procedure
- Half-Mack Sampling

Half-Mack ¡^{2} Analytic Solution

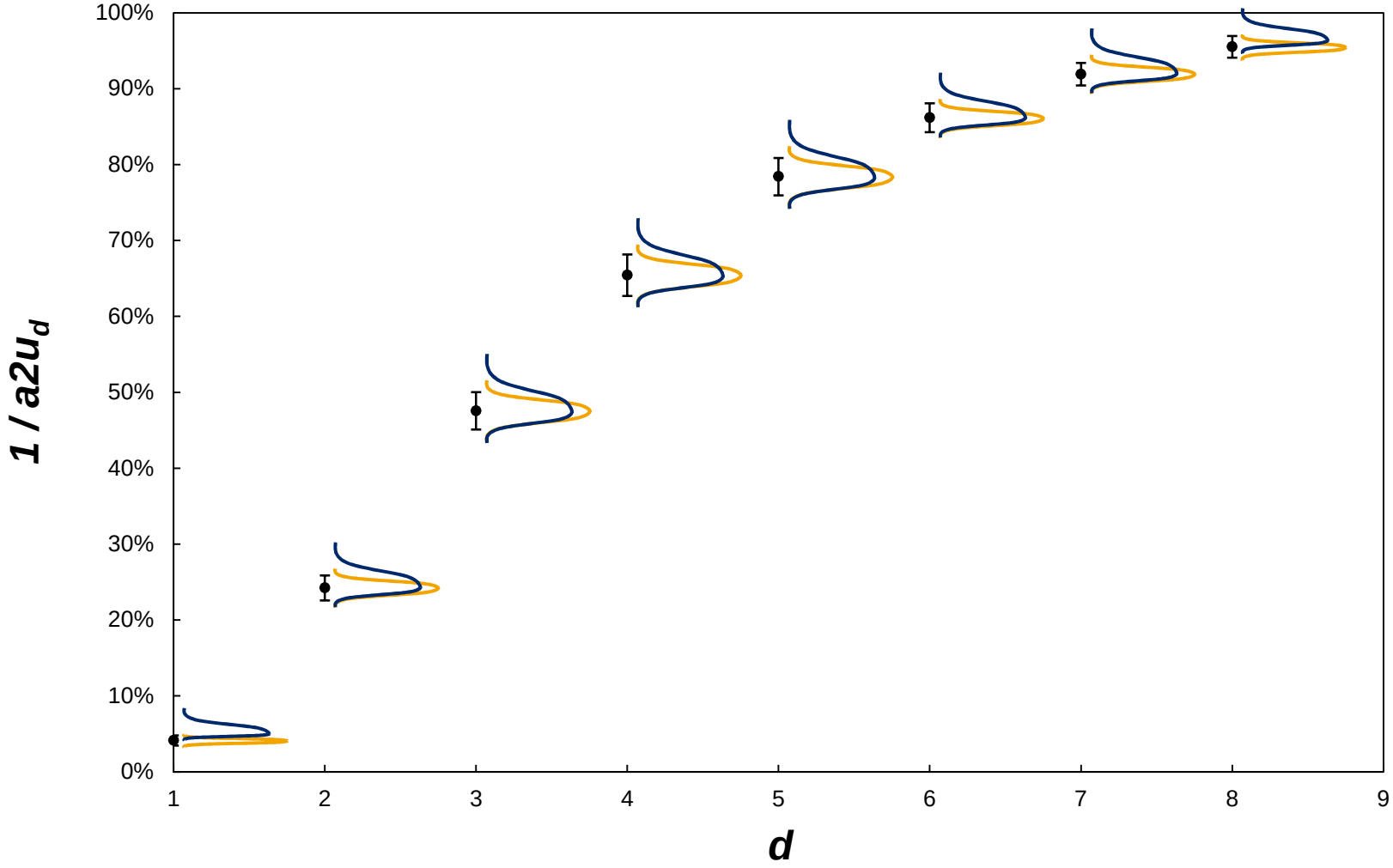
Hypothesis:

$a2u_d$ fluctuate normally
$a2u_d$ fluctuations are independent

Relaxation 1: $a2u_d$ fluctuate with fatter tail

Relaxation 2: $a2u_d$ fluctuate with dependence

Half-Mack Loss Development Patterns

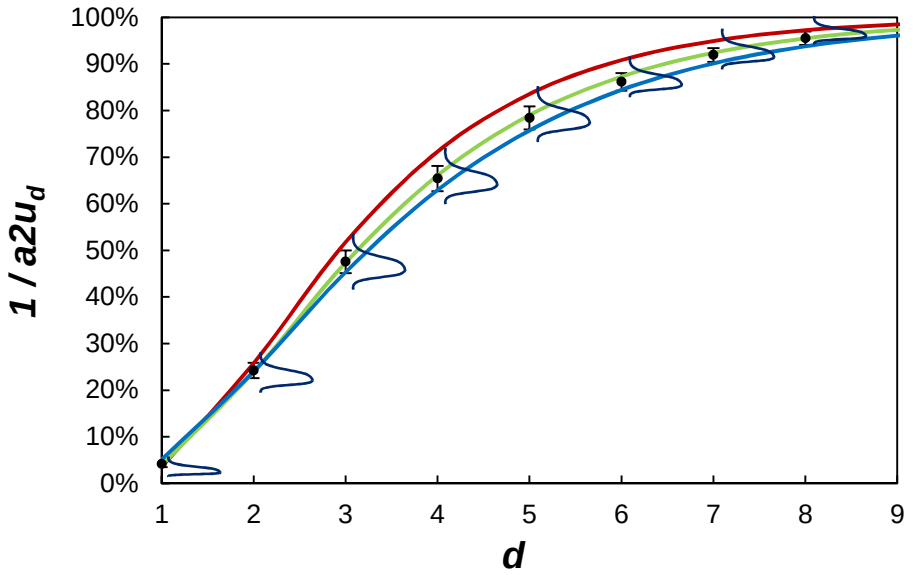


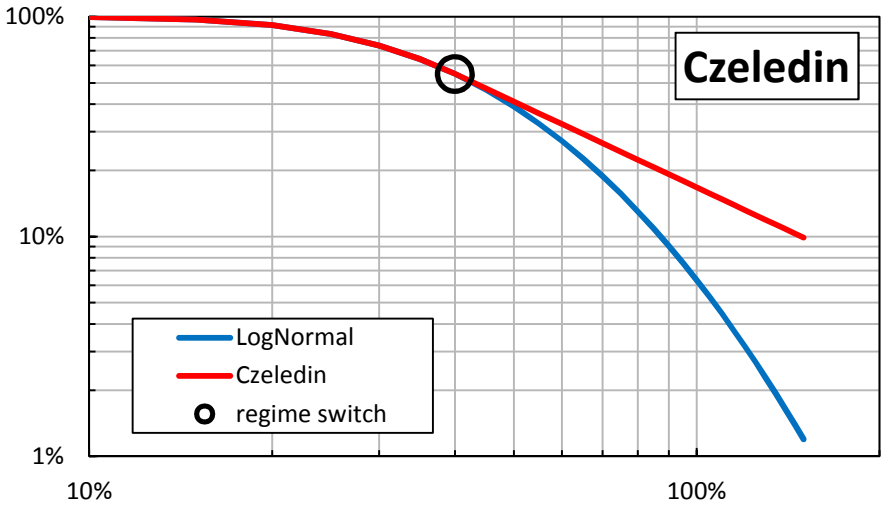
Half-Mack Sampling

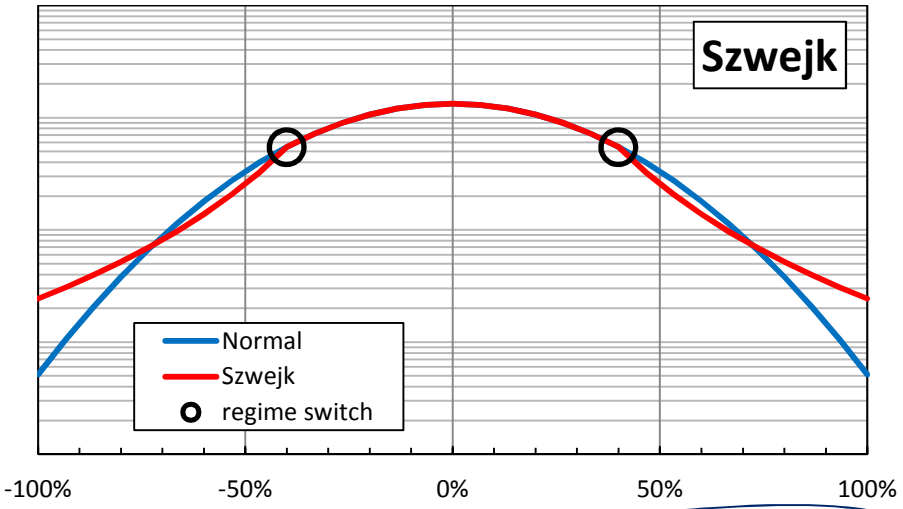
¡^{2} statistic assumes Gaussian errors

$\Rightarrow$ Sample with **fait tails**

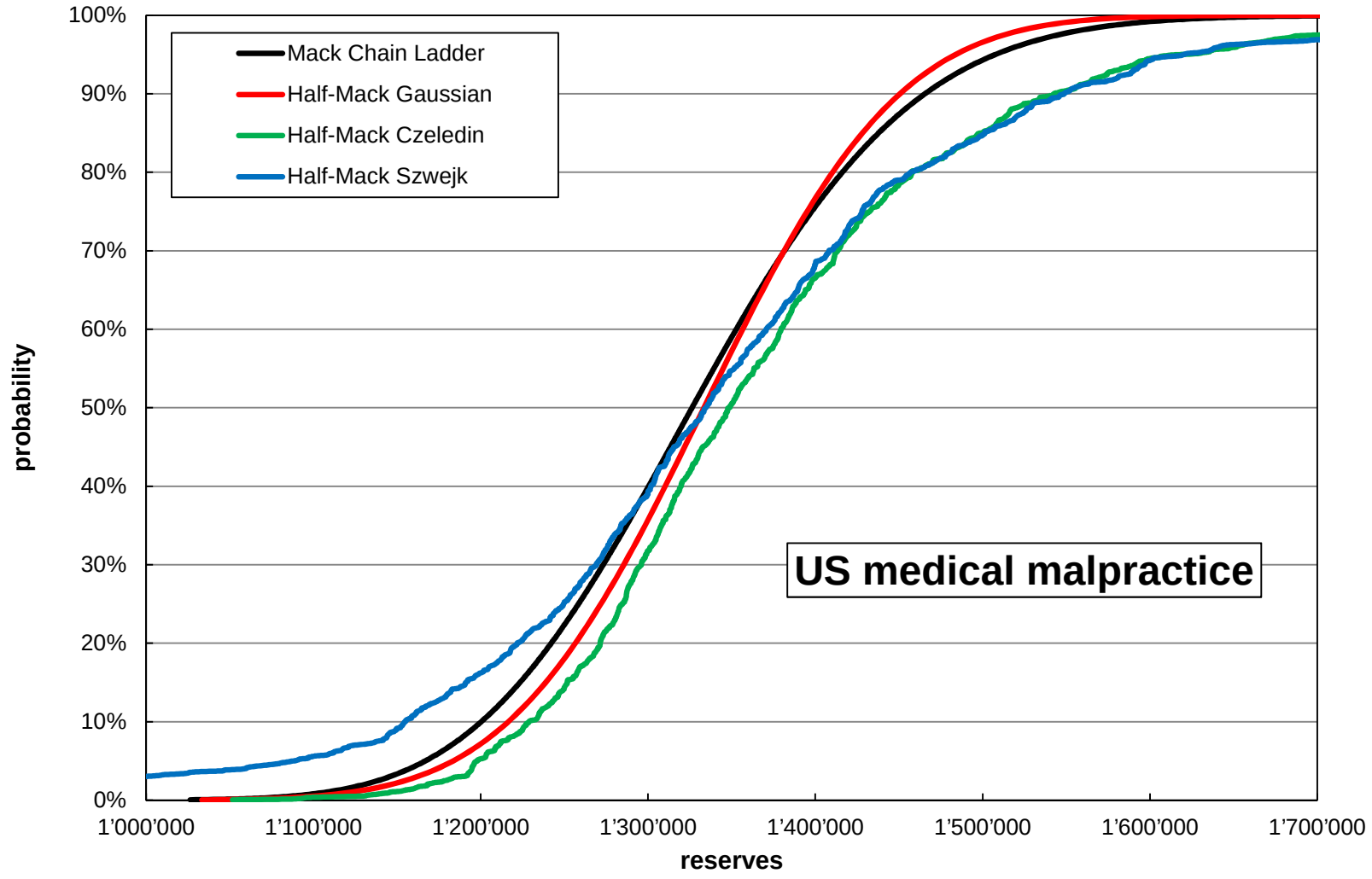
- Sample the $a2u_d$
- Fit the best $F(d)$







Half-Mack Reserves in Comparison



Half-Mack ¡^{2} Analytic Solution

Hypothesis:

$a2u_d$ normally distributed
$a2u_d$ independent

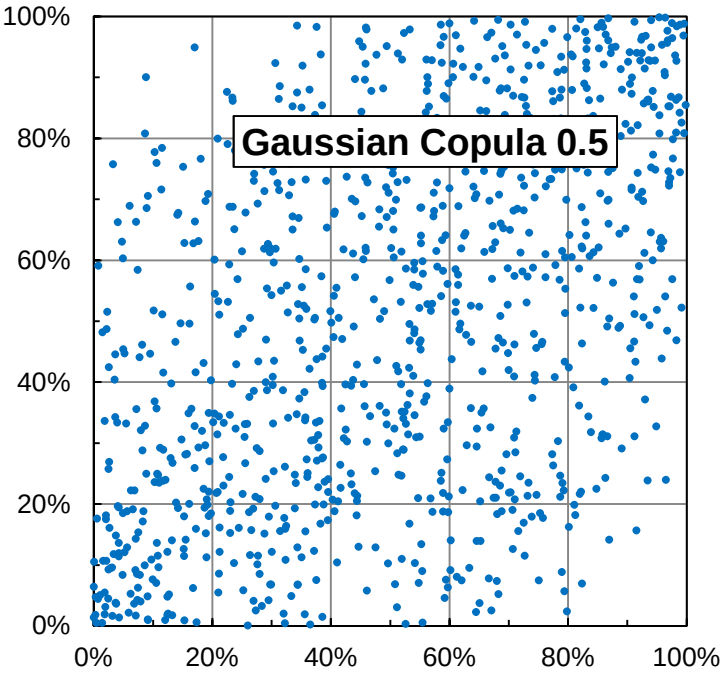
Relaxation 1: $a2u_d$ distributed with fatter tail

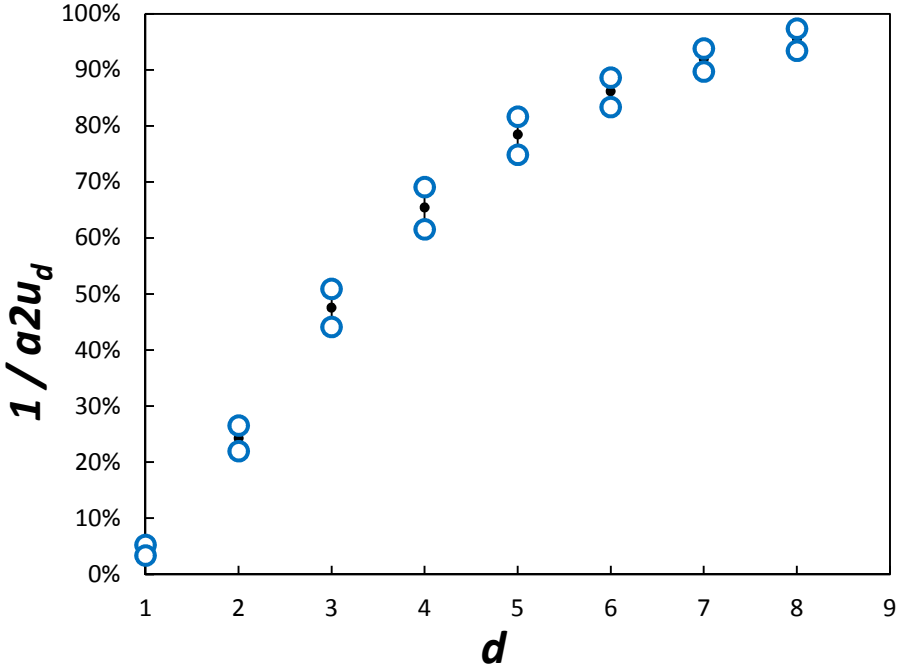
Relaxation 2: $a2u_d$ distributed with dependence

Half-Mack Sampling

¡^{2} statistic assumes independent errors

- Sample the $a2u_d$
- Fit the best $F(d)$

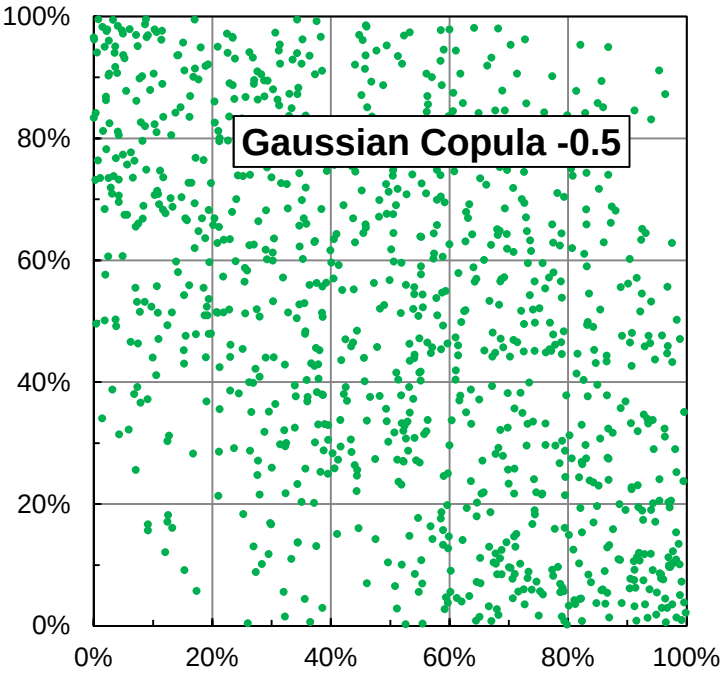


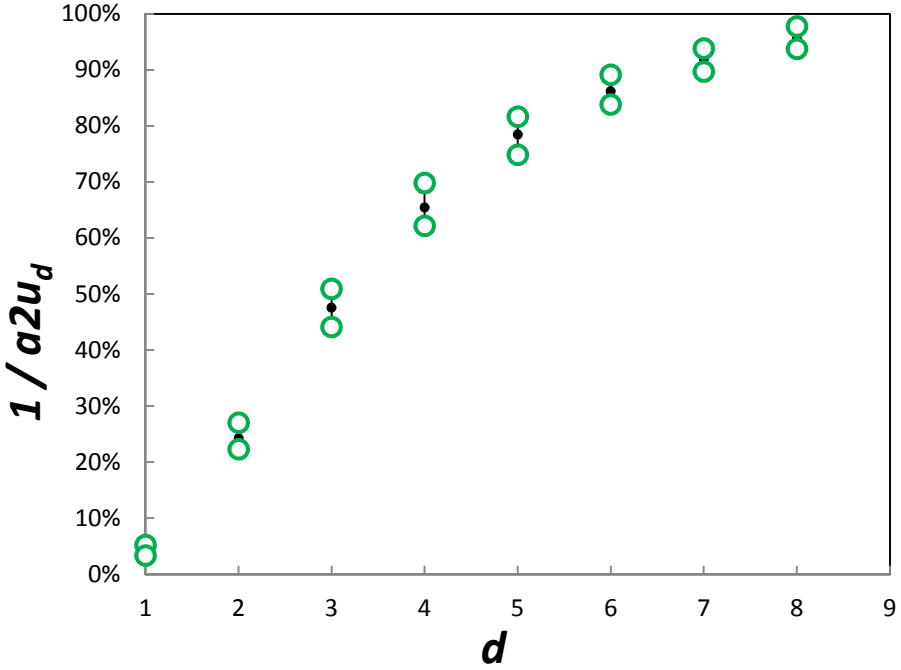


Half-Mack Sampling

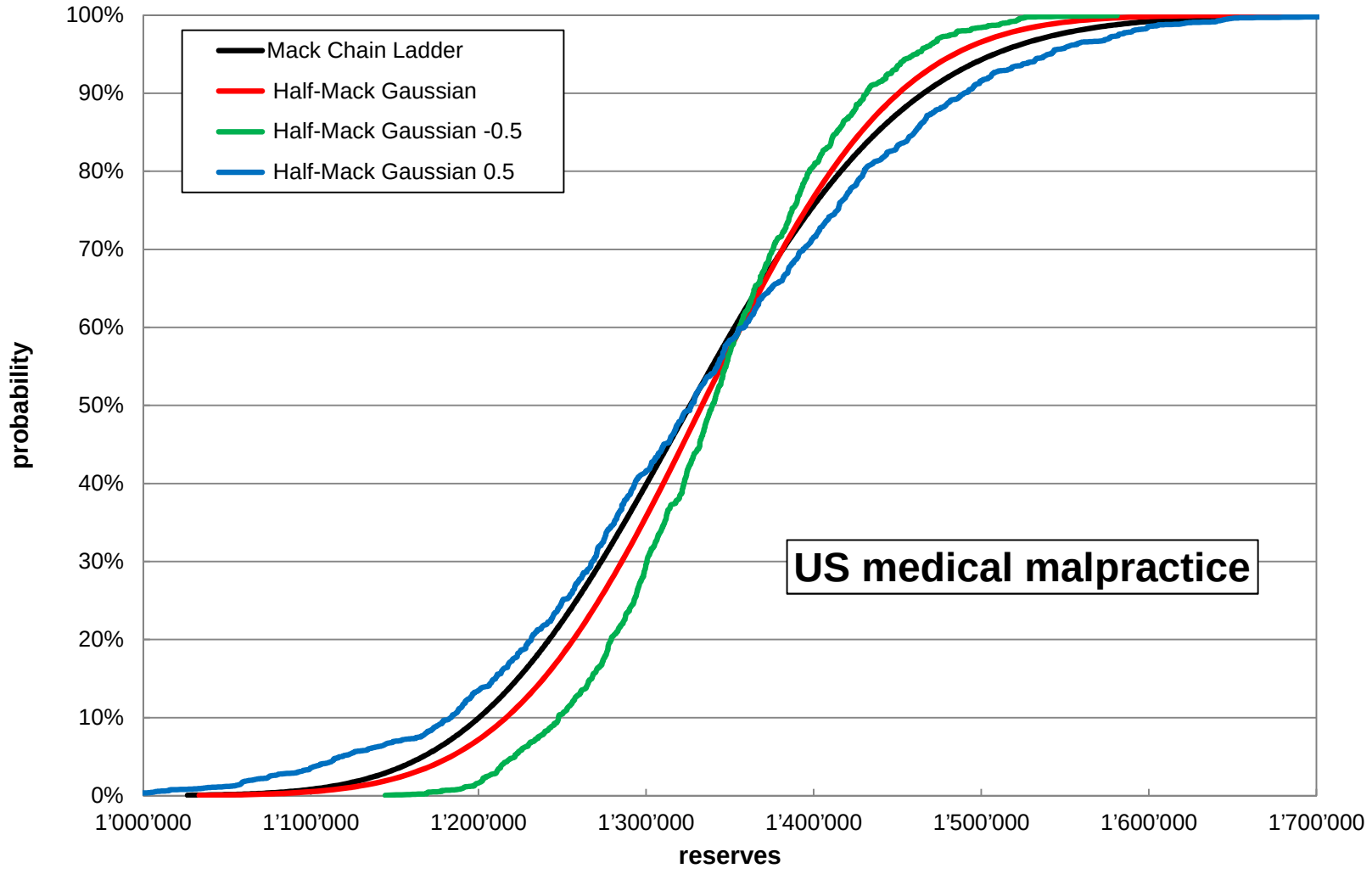
¡^{2} statistic assumes independent errors ⇒ sample with **Gaussian copula**

- Sample the $a2u_d$
- Fit the best $F(d)$





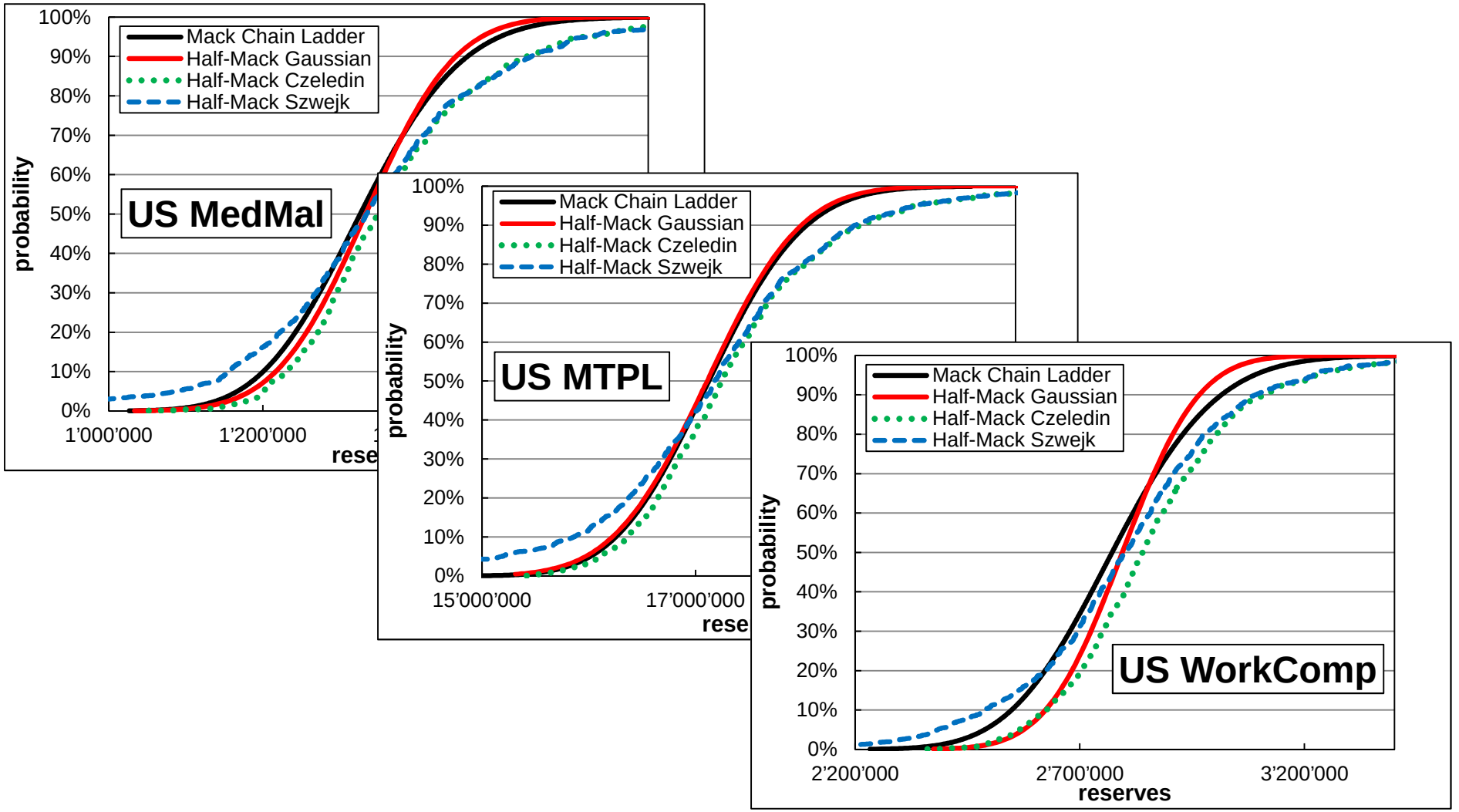
Half-Mack Reserves in Comparison



Agenda

- Mack Chain Ladder Procedure
- Half-Mack Procedure
- Half-Mack Sampling

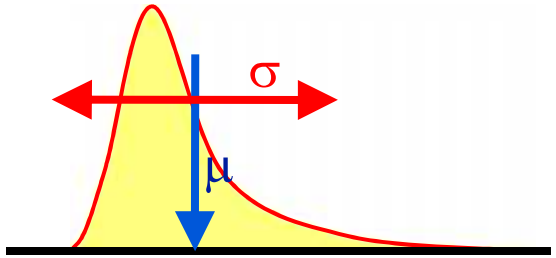
Half-Mack Reserves in Comparison



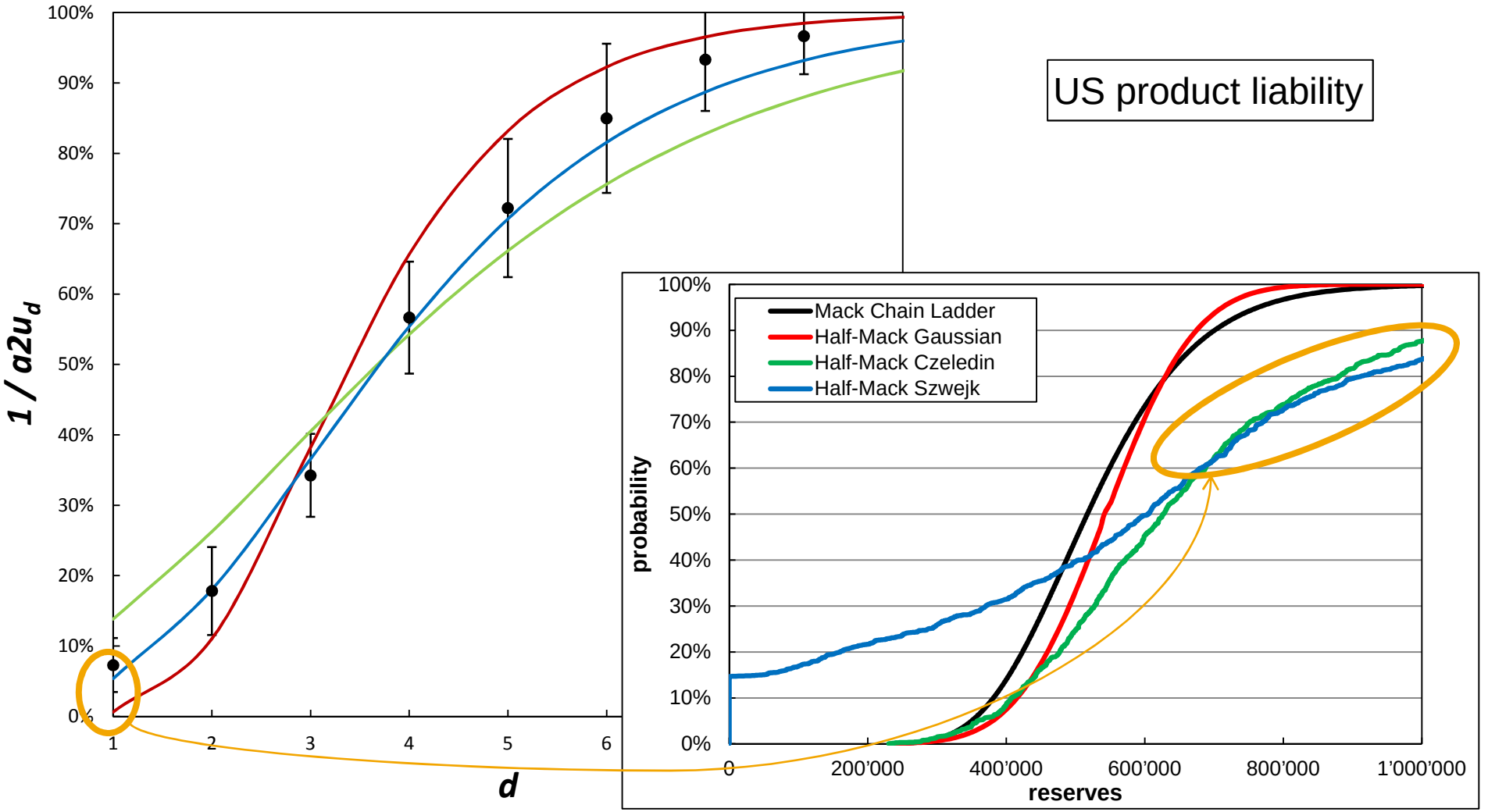
The Half-Mack Chain Ladder Procedure

- Systematic & statistical errors
- Non-parametric
- Natural smoothing of the development factors
- Full distribution of reserves
- Automatic tail factors
- Accounts for market experience

- No smoothing/adjustment of the development factors
- Only 2 moments
- No tail factors



Half-Mack & Actuarial Judgment



US product liability

Conclusion

The Half-Mack procedure is a powerful instrument to account for parameter risk.

The Half-Mack procedure is yet another stochastic reserving tool.

There is no silver bullet: actuarial engineering remains an art ☺

Contact

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