Halb-Mack Stochastische Reservierung

Bahnhofskolloquium

Dezember 2013
### US NAIC Schedule P Data: medical malpractice

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## A Loss Development

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## A Loss Development Triangle

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**Legend:**
- **Accident Years:** 1-10
- **Development Years:** 2-10
- **Calendar Years:** 3-10

**Note:** The data represents the development of losses over time, with each year's losses developing into future years.
## A Loss Development Triangle

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**calendar year 2022**
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**ultimates**
US NAIC Schedule P Data

- http://www.casact.org/research/index.cfm?fa=loss_reserves_data

- Many carriers

- 6 lines of business
  1. personal auto
  2. commercial auto
  3. medical malpractice
  4. workers compensation
  5. general liability
  6. product liability
Agenda

- Mack Chain Ladder Procedure
- Half-Mack Procedure
- Half-Mack Sampling
Chain Ladder Procedure

Notations:

\( a \) = accident year
\( d \) = development year
\( C_{a,d} \) = cumulative losses

Algorithm:

\[
\lambda_{d-1} = \frac{\sum_{a=1}^{n-d+1} C_{a,d}}{\sum_{a=1}^{n-d+1} C_{a,d-1}}
\]

\[
C_{a,d} = \lambda_{d-1} \cdot C_{a,d-1}
\]
Mack Chain Ladder Procedure

Hypothesis:

\[ C_{a,d} \text{ independent for each accident year } a \]

\[
E[C_{a,d}|C_{a,1} \ldots C_{a,d-1}] = \lambda_{d-1} \cdot C_{a,d-1}
\]

\[
V[C_{a,d}|C_{a,1} \ldots C_{a,d-1}] = \sigma_{d-1} \cdot C_{a,d-1}
\]

- Variance proportional to the mean
- Same development pattern for all accident years
- Future depends only on the last diagonal
- Independent accident years
Mack Chain Ladder Procedure

**Hypothesis:**

\[ C_{a,d} \] independent for each accident year \( a \)

\[
E[C_{a,d}|C_{a,1}\cdots C_{a,d-1}] = \lambda_{d-1} \cdot C_{a,d-1}
\]

\[
V[C_{a,d}|C_{a,1}\cdots C_{a,d-1}] = \sigma_{d-1} \cdot C_{a,d-1}
\]

**Consequences:**

\[
\lambda_{d-1} = \frac{\sum_{a=1}^{n-d+1} C_{a,d}}{\sum_{s=1}^{n-d+1} C_{a,d-1}} \quad \rightarrow \quad E[R]
\]

\[
\sigma_{d-1} = \frac{1}{n-d} \sum_{a=1}^{n-d+1} C_{a,d-1} \left( \frac{C_{a,d}}{C_{a,d-1}} - \lambda_{d-1} \right)^2 \quad \rightarrow \quad V[R]
\]

**Application:**

- Assume an underlying distribution of the reserves
- Fit with the first 2 moments
Mack Chain Ladder Procedure

- Systematic & statistical errors
- Non-parametric

- No smoothing/adjustment of the development factors
- No tail factors
- Only 2 moments

![Diagram showing ±μ and ±σ]
Mack Chain Ladder Procedure

**Hypothesis:**

\[ C_{s,d} \text{ independent for each accident year } s \]

\[ E\left[C_{a,d} | C_{a,1} \cdots C_{a,d-1}\right] = \lambda_{d-1} \cdot C_{a,d-1} \]

\[ V\left[C_{a,d} | C_{s,1} \cdots C_{a,d-1}\right] = \sigma_{d-1} \cdot C_{a,d-1} \]

**Consequences:**

\[ \begin{align*}
\lambda_{d-1} &= \frac{\sum_{a=1}^{n-d+1} C_{a,d}}{\sum_{a=1}^{n-d+1} C_{a,d-1}} \quad \rightarrow \quad E[R] = E[a2u_d] \\
\sigma_{d-1} &= \frac{1}{n-d} \sum_{a=1}^{n-d+1} C_{a,d-1} \left(\frac{C_{a,d}}{C_{a,d-1}} - \lambda_{d-1}\right)^2 \quad \rightarrow \quad V[R] = V[a2u_d]
\end{align*} \]

**Application:**

- Assume an underlying distribution of the reserves
- Fit with the first 2 moments
- Take over the first 2 moments only of the development factors
- Fit a model to the development factors
Agenda

- Mack Chain Ladder Procedure
- Half-Mack Procedure
- Half-Mack Sampling
Loss Development Pattern
Loss Development Pattern

Reserving actuaries tricks:
- Adjustments to experience
- Smoothing
- Fit to a model

\[ F(t)^{-1} = \left[ 1 - \exp\left( -\frac{t - \tau}{\lambda} \right) \right]^{-\alpha} \]

- Location
- Shape
- Scale

loss development pattern
loss development factor
development year
continuous time
Loss Development Patterns

![Graph showing the relationship between 1/a2U_d and d. The graph plots a curve that increases as d increases, indicating a non-linear relationship.](image-url)
Half-Mack Loss Development Patterns

Fit with $\chi^2$ statistic

- Best estimate minimizes $\chi^2$ function

CL estimation → model

$$\chi^2 = \sum_{d=1}^{T} \left( \frac{a2u_d - F(d)}{\Delta a2u_d} \right)^2$$

MCL error

- Goodness of fit $\frac{\chi^2_{\text{min}}}{\text{dof}}$
Half-Mack Reserves Distribution

- best fit: $\chi^2 = \chi_{\text{min}}^2$

- $\chi^2$ confidence intervals
  - $1\sigma$ interval: $\chi^2 \leq \chi_{\text{min}}^2 + 1$
  - $p$ intervals: $\chi^2 \leq \chi_{\text{min}}^2 + \chi^2(p)$

→ reserves distribution
The Mack Chain Ladder Procedure

- Systematic & statistical errors
  - Non-parametric
- Natural smoothing of the development factors
- Full distribution of reserves
- Automatic tail factors
- Accounts for market experience

- No smoothing/adjustment of the development factors
- Only 2 moments
- No tail factors

\[ \mu, \sigma \]
The Half-Mack

**Reserving Risk**
- Estimate $a_2u_d$ with errors $\Delta a_2u_d$
- Fit $a_2u_d$ with $F(d)$
- $F(d)$ determines reserves
- $F(d)$ confidence intervals determine reserves distribution

**Parameter Risk in General**
- Estimate $q_x$ with errors $\Delta q_x$
- Fit $q_x$ with $G(x)$
- $G(x)$ determines EV
- $G(x)$ confidence intervals determine EV distribution
Agenda

- Mack Chain Ladder Procedure
- Half-Mack Procedure
- Half-Mack Sampling
**Half-Mack $\chi^2$ Analytic Solution**

**Hypothesis:**
- $a2u_d$ fluctuate normally
- $a2u_d$ fluctuations are independent

**Relaxation 1:**
- $a2u_d$ fluctuate with fatter tail

**Relaxation 2:**
- $a2u_d$ fluctuate with dependence
Half-Mack Loss Development Patterns

![Graph showing loss development patterns with 1/a2ud on the y-axis and d on the x-axis. The graph includes various data points and error bars.](image-url)
\( \chi^2 \) statistic assumes Gaussian errors

\[ \Rightarrow \text{Sample with fault tails} \]

- Sample the \( a2u_d \)
- Fit the best \( F(d) \)
Half-Mack Reserves in Comparison

US medical malpractice
Half-Mack $\chi^2$ Analytic Solution

**Hypothesis:**

\[ a_2 u_d \text{ normally distributed} \]
\[ a_2 u_d \text{ independent} \]

**Relaxation 1:**

\[ a_2 u_d \text{ distributed with fatter tail} \]

**Relaxation 2:**

\[ a_2 u_d \text{ distributed with dependence} \]
Half-Mack Sampling

$\chi^2$ statistic assumes independent errors
- Sample the $a_{2u_d}$
- Fit the best $F(d)$
**Half-Mack Sampling**

\( \chi^2 \) statistic assumes independent errors \( \Rightarrow \) sample with **Gaussian copula**

- Sample the \( a^2 u_d \)
- Fit the best \( F(d) \)
Half-Mack Reserves in Comparison

- Mack Chain Ladder
- Half-Mack Gaussian
- Half-Mack Gaussian -0.5
- Half-Mack Gaussian 0.5

US medical malpractice
Mack Chain Ladder Procedure

Half-Mack Procedure

Half-Mack Sampling
Half-Mack Reserves in Comparison

US MedMal

US MTPL

US WorkComp
The Half-Mack Chain Ladder Procedure

- Systematic & statistical errors
- Non-parametric
- Natural smoothing of the development factors
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- Accounts for market experience

- No smoothing/adjustment of the development factors
- Only 2 moments
- No tail factors
Half-Mack & Actuarial Judgment

US product liability
The Half-Mack procedure is a powerful instrument to account for parameter risk.

The Half-Mack procedure is yet another stochastic reserving tool.

There is no silver bullet: actuarial engineering remains an art 😊
Contact

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