SAV AG SST Nichtlebenmodell
Schätzung der Parameter

Part I
Zusammenfassung

1 Ziel dieses Dokuments

Das vorliegende Dokument verfolgt folgende Ziele:

a) die Herkunft der derzeitigen Standard-Parameterwerte kurz darzulegen,

b) die mathematischen Grundlagen betreffend Parameterschätzer unter Einbezug neuerer Entwicklungen in einem Dokument aufzuschreiben und festzuhalten,

c) die jetzigen Standard-Parameterwerte zu diskutieren,

d) Änderungsvorschläge und Anregungen für mögliche nächste Schritte zu geben.

2 Zur Historie der Parameter-Schätzer im SST

In 2003/2004 wurde das SST Standard-Modell Nichtleben erarbeitet. Charakteristisch sowohl für die Entwicklung des SST in 2003/2004 wie auch die ersten Jahre danach war der Wille zur Zusammenarbeit sowohl zwischen den einzelnen Gesellschaften wie auch zwischen der Aufsichtsbehörde und den Experten in der Versicherungsindustrie.

Nachdem das Standardmodell entwickelt war, mussten für den ersten Field-Test 2004 noch die zugehörigen Parameter geschätzt werden. Dies musste damals in sehr kurzer Zeit geschehen. Auch dies war eine Gemeinschaftsarbeit, bei der die SAV und die Versicherungsindustrie vertreten durch actuaries aus der Praxis sowie das damalige BPV involviert waren.

Die Korrelationsmatrizen wurden rein "Bayesianisch" ermittelt, d.h. man hat sich überlegt, welche Branchen korreliert sein könnten, und hat dann eine grobe Einteilung (keine Korrelation, Korrelationen von 0.25 und 0.5) vorgenommen.

Die Pareto-Parameter für die Einzel-Grossschäden wurden von mehreren grösseren Gesellschaften mit maximum likelihood geschätzt. Die einzelnen Gesellschaften meldeten ihre Resultate an das BPV, welches dann aufgrund dieser Umfrage die "Standard-Parameter" festlegte. Auch die Standardfaktoren für den Anteil der Einzel-Grossschäden wurde via Umfrage ermittelt. Die Parameter für Kumulgrossschäden wurde aufgrund spezieller Überlegungen bestimmt.

Analog wurde beim Zufallsrisiko CY vorgegangen. Hier wurden die Variationskoeffizienten der Einzelschadenhöhen von mehreren Gesellschaften ermittelt und an das BPV gemeldet, welches dann die Standard-Werte festlegte.

Für das Parameterrisiko gab es noch keine mathematisch fundierte Theorie. Hier ist man pragmatisch vorgegangen. Beim Parameter-Risiko CY hat man sich bei den Berechnungen wo immer möglich auf Gemeinschaftsstatistiken abgestützt und vereinfachend angenommen, dass angesichts des grossen Volumens das Zufallsrisiko in den Zahlen der Gemeinschaftsstatistik vernachlässigt werden kann. Beim Parameter-Risiko für das Abwicklungsergebnis (Risiko PY) wurden Berechnungen mit der Mack-Formel für das ultimate Reserve-Risiko vorgenommen und daraus die Werte abgeleitet. Für das Zufallsrisiko gibt es keine Standardwerte und die Gesellschaften wurden angehalten, dies aufgrund der gesellschaftseigenen beobachteten Abwicklungsergebnisse in vergangenen Jahren abzuschätzen.

Mittels Umfrage bei den Gesellschaften wurden vom BPV auch die Default-Zahlungspattern für die einzelnen Branchen ermittelt.

Aus Zeitgründen wurde damals in 2004 auf eine weitergehende Dokumentation bezüglich der Schätzung der Parameter verzichtet.

Die Modellierung von UVG mit dem Replikationsportfolio erfolgte etwas später und war das Resultat von Diskussionen innerhalb einer vom BPV geführten Arbeitsgruppe. Kurz zusammengefasst stecken folgende Überlegungen dahinter: die Finanzierung des Teuerungsteils auf den UVG-Renten ist durch die gesetzliche Grundlage zur Erhebung von Umlagebeiträgen und den Pool-Mechanismus abgesichert, so dass dafür keine zusätzlichen Rückstellungen (ausserhalb des Betrags im UVG-Pool zur Sicherung der indexierten Renten) und kein zusätzliches Risikokapital erforderlich sind. Mit der UVG-Regelung ist das Renten-Deckungskapital (inklusive Teil im UVG-Pool) mit dem 10-jährigen Durchschnitt der 10-jährigen Bundesobligationen zu bedienen. Unter der vereinfachenden Annahme, dass dieses Kapital konstant bleibt über die Zeit, generiert ein Portfolio bestehend aus 10 gleichen Tranchen von 10-jährigen Bundesobligationen automatisch die Verzinsungs-Verpflichtungen und ist somit ein replizierendes Portfolio für diese Verpflichtungen. Die vereinfachende Annahme eines konstanten Kapitals trifft natürlich nicht voll zu, doch ist man zum Schluss gekommen, dass der Fehler nicht gross ist und dass keine praktikable und bessere Alternativmethode zur Verfügung steht.

Auch die Parameter und die Modellierung von ES wurde aufgrund zwischenzeitlich erfolgter Tarifberechnungen aktualisiert und die bei diesen Berechnungen verwendete Modellierung übernommen.

Ansonsten sind die damals in 2004 ermittelten und festgelegten "Standard-Parameterwerte" unverändert beibehalten worden und entsprechen heute noch den im FINMA-Template erscheinenden Default Standard-Parameterwerten.

3 Zwischenzeitlich erfolgte theoretische Entwicklungen

In der Zwischenzeit gab es verschiedene Weiterentwicklungen auf theoretischem Gebiet. So ist insbesondere eine Formel für das einjährige Abwicklungsrisiko für das chain-ladder Reservierungsmodell entwickelt worden (Merz-Wüthrich [10], aber auch Gisler-Wüthrich [5] sowie Bühlmann & alias [2]). In [5] wird auch gezeigt, wie Credibility zur Schätzung der Abwicklungsfaktoren benutzt werden kann.

In der Masterarbeit von Gradenwitz [6] und in einer nachfolgenden Arbeit von Gisler [4] für das ASTIN-Kolloquium 2009 in Helsinki ("Helsinki-Arbeit") wird das SST-Standard-Modell Nichtleben mit dem Solvency II Modell verglichen und auch auf die Schätzung der Parameter eingegangen. Insbesondere wurden dort Schätzer für die Parameter-Risiken basierend auf einem soliden mathematischen Fundament entwickelt.

4 Zusammenfassung der Ergebnisse

Die diesem Dokument zugrundeliegenden mathematischen Grundlagen wurden in [4] publiziert und sind hier als Teil II beigefügt. In diesem Abschnitt werden die wesentlichen Ergebnisse zusammengefasst.

4.1 CY-Risiken

4.1.1 Zufallsrisiko CY (Normalschäden)

Dazu müssen einerseits die Variationskoeffizienten der Einzelschadenhöhen sowie die erwartete Anzahl Schäden geschätzt werden. Letzteres wird hier nicht kommentiert, da eine Prognose der Schadenanzahl in jeder Geselleschaft im Rahmen des Budgeting Prozesses und des Pricing vorhanden sein sollte.

Die Variationskoeffizienten können ganz einfach mittels der Formel (25) geschätzt werden. Der einzige zu beachtende Punkt ist, dass sich die Verteilung der Einzelschadenhöhen im Laufe der Abwicklung verändert und die Varianz in der Regel grösser wird (Grossschäden wachsen erst im Laufe der Abwicklung in die höheren Layer hinein). Die mit Formel (25) geschätzten Variationskoeffizienten weisen daher bei Long-Tail Branchen für die neueren und noch wenig abgewickelten Schadenjahre einen negativen Bias auf. Dies sollte bei der Schätzung berücksichtigt werden, indem entweder die erhaltenen Variationskoeffizienten für die neueren Jahre extrapoliert werden (z.B. mit chain ladder Prognose wie dargelegt in Abschnitt B.1) oder indem man die Werte für die neuesten zwei bis drei Jahre einfach weglässt und ein Mittel über den Rest der Jahre nimmt.

Die Variationskoeffizienten der einzelnen Branchen sind in der Regel recht stabil über die Zeit. Dieser Parameter lässt sich meist auch von mittleren und kleineren Gesellschaften aufgrund der eigenen Daten ermitteln.

4.1.2 Parameterrisiko CY (Normalschäden)

Das Parameter-Risiko kann mit der Formel (28) geschätzt werden. Dieser Schätzer ist mathematisch sauber abgestützt und basiert auf dem SST zugrundeliegenden Credibility

Modell.

Gemäss dem Credibility Modell hat das Parameter-Risiko eine spezielle Struktur und kann auch geschrieben werden als

$$\sigma^2_{param} \simeq \mathrm{Var}\left(\Theta_1\right) + \mathrm{Var}\left(\Theta_2\right), \tag{1}$$

wobei der erste Term die Varianz des Erwartungswertes der Schadenanzahl und der zweite derjenige des Erwartungswertes der Schadenhöhe wiederspiegelt. Diese spezielle Struktur wird im oben genannten Schätzer (28) nicht berücksichtigt.

An Stelle oder in Ergänzung von (28) kann auch (1) verwendet und die dort erscheinenden Komponenten$\mathrm{Var}\left(\Theta_1\right)$ und $\mathrm{Var}\left(\Theta_2\right)$ direkt geschätzt werden. Dieses Vorgehen ist in Branchen, wo die Jahresrisiken ein "natürliches Volumenmass" bezüglich Frequenz darstellen (z.B. MFH, Hausrat, jedoch nicht KKV, UVG, AH) angebracht. Der erste Term, $\mathrm{Var}\left(\Theta_1\right)$ kann dann mit Formel (32) geschätzt werden. Die Schätzung von $\mathrm{Var}\left(\Theta_2\right)$ ist etwas komplizierter (siehe Literaturhinweis in Abschnitt B.2). Oft wird der Schätzer einen Wert in der Nähe von Null oder Null ergeben, da die Variabilität in den Schadenhöhen auch bei bekanntem Risikoparameter Θ_2 recht hoch ist und diese eine allfällige zugrundeliegende Variabilität in den "wahren Erwartungswerten" dominiert. Der bedeutendere Term wird in der Regel $\mathrm{Var}\left(\Theta_1\right)$ sein, so dass die Vernachlässigung von $\mathrm{Var}\left(\Theta_2\right)$ oder eine Bayesianische Schätzung von $\mathrm{Var}\left(\Theta_2\right)$ aufgrund von a priori Überlegungen (z.B. wie genau ist die Schätzung der Schadenteuerung (Variationskoeffizient)) in vielen Fällen angebracht und sinnvoll ist.

Als weitere Bemerkung sei angefügt, dass bei kleineren Gesellschaften die Schätzung des Parameter-Risikos aufgrund der eigenen Daten zu wenig aussagekräftig ist und die Verwendung der Standardwerte angezeigt ist. Allerdings wäre eine Überprüfung und Neuschätzung dieser Standardwerte durch die FINMA mehr als angebracht, insbesondere, da diese Werte damals sehr schnell und rudimentär festgelegt wurden.

4.1.3 Schätzung des Pareto-Parameters für die Schadenhöhe der Grossschäden

Der Pareto-Parameter für Pareto-verteilte Grossschäden wird klassisch mit Maximum-Likelihood geschätzt (siehe Formel (33)). In vielen Branchen ist jedoch die Anzahl der beobachteten Einzel-Grossschäden auch bei grösseren Gesellschaften relativ klein, so dass der ML-Schätzer eine erheblich Schätzungenauigkeit aufweist.

Doch gibt es eine gute mathematische Theorie, wie dieser Parameter mittels Credibility geschätzt werden kann (siehe Formel (35)). Die Verwendung dieses Schätzers ist nicht nur für kleine, sondern auch mittlere oder sogar grössere Gesellschaften sinnvoll.

Auch hier ist anzufügen, dass ein update der Default-Werte durch die FINMA mittels Umfrage bei den Gesellschaften angezeigt wäre. Mit den Daten dieses updates könnte auch der in (35) erscheinende Parameter κ ermittelt werden.

4.2 PY-Risiken

Für das einjährige Abwicklungsrisiko gibt es die Formel von Merz-Wüthrich für das chain-ladder Modell (Formeln (38) und (39)), die hier verwendet werden kann. Weitere Forschungen sind im Gange, und Resultate für das einjährige Abwicklungsrisiko für weitere Reservierungsmodelle sind in den nächsten Jahren zu erwarten. Es lohnt sich, die Literatur hier etwas zu verfolgen.

Es ist absolut zwingend und wichtig zu sehen, dass es beim SST um das Risiko eines "Jahrhundert-Abwicklungsverlustes" geht. Solche "Jahrhundert-Ereignisse" fehlen in der Regel in den beobachteten Abwicklungsdreiecken. Das Reserve-Risiko wird daher unterschätzt, wenn man es einzig mit einem analytischen Modell und der Formel von Merz-Wüthrich ermittelt. Es ist zwingend zu ergänzen durch Szenario-Betrachtungen (siehe dazu auch die Ausführungen in Abschnitt B.4).

Für kleinere Gesellschaften besteht zudem die Schwierigkeit, dass die beobachteten chain-ladder Faktoren sehr volatil sein können. Auch hier könnte man mit Credibility-Theorie aus den beobachteten individuellen Daten und Industrie-weiten Daten eine bessere Schätzung für das Abwicklungspattern und für das Reserve-Risiko erhalten. Die Theorie dazu existiert (siehe [5]). Aufgrund der Beobachtungen von mehreren Gesellschaften könnten nicht nur die a priori zu erwartenden Abwicklungsmuster, sondern auch die für Credibility benötigten Parameterwerte aufgrund geschätzt werden. Dies würde jedoch eine Zusammenarbeit bzw. eine Erfassung von Abwicklungsdreiecken von mehreren Gesellschaften erfordern.

4.3 Korrelationen

Wie in Abschnitt A.3 dargelegt, sind konzeptionell drei Korrelationsmatrizen zu bestimmen:

$\mathbf{R}_{CY}$, $\mathbf{R}_{PY}$, $\mathbf{R}_{CY,PY}$ (für CY, PY, sowie Korrelationen zwischen CY und PY). Das ist auch mit den derzeitigen Standard-Parameter für die Korrelationsmatrizen so, doch ist mit den heutigen Werten $\mathbf{R}_{PY}$ die Identitätsmatrix, während $\mathbf{R}_{CY,PY}$ aus lauter Nullen besteht.

Bei der Festsetzung dieser Werte in 2004 war man sich der Wirkung von Kalenderjahr-Effekten zu wenig bewusst. Solche Kalender-Jahr Effekte (z.B. Inflation) induzieren sowohl eine Korrelation des Abwicklungsergebnisses zwischen verschiedenen Branchen wie auch eine Korrelation zwischen Schadenaufwand GJ und Abwicklungsergebnissen, sprich CY- und PY Risiken. Die zur Zeit gültigen Standardwerte für die Korrelationsmatrizen geben daher ein zu optimistisches Bild ab. Das versicherungstechnische Risiko wird damit stark unterschätzt. Zu beachten ist auch, dass in Solvency II bedeutend höhere Korrelationen verwendet werden.

Die Bestimmung der Korrelationsmatrizen mit quantitativen Methoden ist allerdings nicht ganz einfach. Ein mögliches Vorgehen mittels eines "Risk-Driver" Ansatzes ist in Abschnitt B.5 dargelegt.

Fest steht jedoch, dass die derzeitigen Standard-Parameter für die Korrelationsmatrix zu optimistisch sind. Eine Überarbeitung ist nicht nur angezeigt, sondern zwingend nötig. Hier besteht nach Meinung der AG Handlungsbedarf von Seiten der FINMA. Die AG ist auch der Meinung, dass ein VA im Wissen um diese Mängel ab sofort diese zu "optimistischen" Default-Korrelationsannahmen nicht mehr verwenden sollte und etwas vorsichtigere Annahmen (z.B. etwas in Richtung Solvency II) verwenden sollte.

Part II

Parameter estimation in the SST: an extract from the "Helsinki Paper"

Abstract:
This part is mainly an extract of [4] (paper presented at the ASTIN Colloquium 2009 in Helsinki). It concentrates on the part of that paper with regard to the parameter estimation and consists of two parts:
part A: modelling,
part B: the estimation of the corresponding parameters.

A Modelling in the SST

In the following we briefly summarise some of the model assumptions used in the SST

A.1 Modelling of the normal claim amount current year

Model Assumptions A.1 *For each line of business i it is assumed that the current year (=next coming year) is characterised by its risk characteristics $\mathbf{\Theta}_i^T = (\Theta_{1i}, \Theta_{2i})$, where Θ_{1i} and Θ_{2i} are independent with $E[\Theta_{1i}] = E[\Theta_{2i}] = 1$, and that conditional on $\mathbf{\Theta}_i^T = (\Theta_{1i}, \Theta_{2i})$*

- *the normal claim amount $C_i^{CY,n}$ is compound Poisson distributed*
- *with Poisson parameter $\lambda(w_i, \Theta_{1i}) = w_i \lambda_i \Theta_{1i}$, where w_i is a known weight and where λ_i is the a priori expected claim frequency,*
- *and with claim severities $Y_i^{(\nu)}$ having the same distribution as $\Theta_{2i} Y_i$, where Y_i has a distribution $F_i(y)$ with $E[Y_i] = \mu_i$.*

Remarks:

- The underlying risk characteristics $\mathbf{\Theta}_i^T = (\Theta_{1i}, \Theta_{2i})$ reflect the "state of the nature" in the next coming year for lob i. Indeed things like weather conditions, economic situation, change in legislation, etc. might have an impact on the claim number and the claim severity. These "conditions" may vary from year to year and the imposed changes increase the risk and affect big companies as much as small companies, i.e. this risk cannot be diversified. The "true claim frequency" in the coming year will be $\lambda_i(\Theta_{1i}) = \lambda_i \cdot \Theta_{1i}$, hence Θ_{1i} is the random factor by which next year's "true underlying claim frequency" will deviate from the a priori expected claim frequency λ_i. The interpretation for Θ_{2i} is quite analogous, but with respect to the "true underlying expected value of the claim severity".

- Note, that no distributional assumption about the claim severity is made in model assumptions A.1. But it will later be assumed that

$$C_\bullet = C_\bullet^{CY,n} + C_\bullet^{PY}$$

can be approximated by a lognormal distribution with the corresponding first and second moments (see Section A.3). Another idea would be to assume a lognormal distribution on the level of lines of business. This would be a slightly different model, since the sum of lognormal distributions is not lognormal any more. But the difference would presumably be rather small without a significant impact on the result, but it would have the disadvantage to be much more complicated and the distribution of $C_\bullet$ could not be expressed by a simple analytical formula any more.

We introduce

$$\widetilde{P}_i = E\left[C_i^{CY,n}\right],$$

which is the pure risk premium for normal claims, and

$$X_i = \frac{C_i^{CY,n}}{\widetilde{P}_i} \tag{2}$$

the corresponding loss ratio.

One can show that with model assumptions A.1

$$\sigma_i^2 := \mathrm{Var}\left(X_i\right) = \sigma_{i,param}^2 + \frac{\sigma_{i,fluct}^2}{\nu_i}, \tag{3}$$

where

$$\begin{aligned} \sigma_{i,param}^2 &= \mathrm{Var}\left(\Theta_{1i}\right) + \mathrm{Var}\left(\Theta_{2i}\right) + \mathrm{Var}\left(\Theta_{1i}\right) \cdot \mathrm{Var}\left(\Theta_{2i}\right) && (4) \\ &\simeq \mathrm{Var}\left(\Theta_{1i}\right) + \mathrm{Var}\left(\Theta_{2i}\right), && (5) \\ \sigma_{i,fluct}^2 &= CoVa^2\left(Y_i^{(v)}\right) + 1. && (6) \end{aligned}$$

and where

$$\begin{aligned} CoVa\left(Y_i^{(v)}\right) &= \text{the coefficient of variation of the claim severity,} \\ \nu_i = w_i\lambda^{(i)} &= \text{a prori expected number of claims.} \end{aligned}$$

Since $\widetilde{P}_i = \nu_i\mu_i$ we can also write (3) as

$$\sigma_i^2 := \mathrm{Var}\left(X_i\right) = \sigma_{i,param}^2 + \frac{\widetilde{\sigma}_{i,fluct}^2}{\widetilde{P}_i}, \tag{7}$$

where

$$\widetilde{\sigma}_{i,fluct}^2 = \mu_i\sigma_{i,fluct}^2.$$

Note that $\sigma_i^2 = CoVa^2\left(C_i^{CY,n}\right)$ is composed of two components, the *parameter risk* and the *random fluctuation risk*. The parameter risk is independent of the size of the company whereas the fluctuation risk decreases with the size of the company respectively with the number of a priori expected claims.

Remark:

- There are *standard values* provided by the supervision authority for $\sigma^2_{i,param}$ and $\sigma^2_{i,fluct}$. Companies might deviate from them based on estimates from their own data. Usually there is sufficient data in most of the companies to estimate $\sigma^2_{i,fluct}$, but there has not been so far a sound methodology to estimate $\sigma^2_{i,param}$ from the own data. Therefore most companies use the default values for $\sigma^2_{i,param}$. In Section B we fill the gap and derive new estimators for estimating $\sigma^2_{i,param}$ from the company-own data.

For calculating the variance of the total normal claim amount (summed up over all lines of business) we have to make assumptions on the correlation of the normal claim amount of different lines of business. Let

$$\begin{aligned}\rho_{ij}^{CY} &:= \mathrm{Corr}\left(C_i^{CY,n}, C_j^{CY,n}\right),\\ X_\bullet &:= \frac{C_\bullet^{CY,n}}{\widetilde{P}_\bullet}.\end{aligned}$$

Then we obtain

$$\mathrm{Var}\left(C_\bullet^{CY,n}\right) = \sum_{i,j=1}^{I} \widetilde{P}_i\,\sigma_i \widetilde{P}_j\,\sigma_j \rho_{ij}^{CY} \tag{8}$$

and

$$\sigma^2 := \mathrm{Var}\left(X_\bullet\right) = \frac{1}{\widetilde{P}_\bullet^2} \sum_{i,j=1}^{I} \widetilde{P}_i\,\sigma_i \widetilde{P}_j\,\sigma_j \rho_{ij}^{CY}. \tag{9}$$

It is convenient to write (8) in matrix notation. Let

$$\begin{aligned}\mathbf{X} &= \left(X_1, X_2, \ldots, X_I\right)^T,\\ \mathbf{W}_{CY} &= \left(\widetilde{P_1}\sigma_1, \widetilde{P_2}\sigma_2, \ldots, \widetilde{P_I}\sigma_I\right)^T,\\ \mathbf{R}_{CY} &= \mathrm{Corr}\left(\mathbf{X}\ ,\ \mathbf{X}^T\right).\end{aligned}$$

Note that $\mathbf{R}_{CY}$ denotes the correlation matrix of $\mathbf{X}$ with the entries

$$\mathbf{R}_{CY}\left(i,j\right) = \rho_{ij}^{CY}.$$

(8) written in matrix notation becomes

$$\mathrm{Var}\left(C_\bullet^{CY,n}\right) = \mathbf{W}_{CY}^T \cdot \mathbf{R}_{CY} \cdot \mathbf{W}_{CY}. \tag{10}$$

A.2 Modelling of the Reserve Risk

Let

$$\begin{aligned}L_i &= \text{outstanding claims liabilities at 1.1. for lob } i,\\ R_i &= \text{best estimate of } L_i \text{ per 1.1.} = \text{best estimate reserve},\\ \widetilde{R_i} &= PA_i^{PY} + R_i^{31.12.,PY} = \text{best estimate of } L_i \text{ per 31.12.},\end{aligned}$$

where PA_i^{PY} are the claim payments for previous years' claims and $R_i^{31.12.,PY}$ are best estimate reserves per 31.12. of previous years' claims

Note that R_i is known at the beginning of the year, whereas $\widetilde{R_i}$ is still a random variable (PA_i^{PY} and $R_i^{31.12.,PY}$ will only be known at the end of the year).

The claim amount PY becomes

$$C_i^{PY} = \widetilde{R_i} - R_i.$$

As for the current year risk, we introduce something analogous to the loss ratio, but with the ingoing reserves R_i (instead of the premiums) as "weight", that is we consider

$$Y_i = \frac{\widetilde{R_i}}{R_i}.$$

Note that $\mathrm{E}[Y_i] = 0$ and that $C_i^{PY} = (Y_i - 1)R_i$.

Model Assumptions A.2 (reserve risk SST) *It is assumed that*

$$\tau_i^2 := \mathrm{Var}\,(Y_i) = \tau_{i,param}^2 + \frac{\tau_{i,fluct}^2}{R_i}. \tag{11}$$

Remarks:

- The SST distinguishes between parameter risk and random fluctuation risk, but the model assumptions A.2 are not written down anywhere in an official document of the SST. As pointed out in [8] the question of how to quantify the reserve is risk is not yet fully answered. Thus the variance structure (11) is my own interpretation and is not yet an integral part of the current SST. I think that from a modelling point of view there are good reasons to assume the above structure, even if this structure is not reflected in the current standard parameters, where the coefficients of variation of the random fluctuation risk do not depend on the size of the reserves. Note that (11) has the same structure as (7).

- As for the current year risk it is again assumed that the variance consists of two components, the parameter risk independent of the weight (=volume of reserves) and the random fluctuation risk which is inversely proportional to this weight. The parameter risk reflects the estimation error which is a risk measure of the deviation of the reserve R_i from the true expected value $E\left[L_i\right]$ of the outstanding liabilities L_i, whereas the random fluctuation risk encompasses the pure random fluctuation of the ultimate around $E\left[L_i\right]$, which is also called *process error* in claims reserving.

In order to calculate the variance for the reserve risk summed up over all lines of business, we have again to make assumptions about the correlation of the reserve risk of different lob. According to the current SST standard assumption it is assumed that there is no correlation between the reserve risk of different lob.

Discussion and remarks on the correlation assumption for PY risks in the SST

- The current standard assumption of no-correlation between the reserve risks of different lines of business is questionable. Calendar year factors may affect the reserves of several lines of business simultaneously and impose a positive correlation. An obvious calendar year factor is certainly inflation, but there might also be others like change in legislation.
- A change of the correlation assumption can be done well within the SST modelling frame-work. Hence the no-correlation assumption does not question the SST-standard-model as such, but is rather a matter of the parameter choice within the SST standard model.

Let

$$\begin{aligned} Y_\bullet &= \frac{\widetilde{R_\bullet}}{R_\bullet}, \\ \mathbf{Y} &= (Y_1, Y_2, \ldots, Y_I)^T, \\ \mathbf{W}_{PY} &= (R_1\tau_1, R_2\tau_2, \ldots, R_I\tau_I)^T, \\ \mathbf{R}_{PY} &= \mathrm{Corr}\left(\mathbf{Y}\ ,\ \mathbf{Y}^T\right). \end{aligned}$$

Then we obtain

$$\mathrm{Var}\left(C_\bullet^{PY}\right) = \mathbf{W}_{PY}^T \cdot \mathbf{R}_{PY} \cdot \mathbf{W}_{PY}, \tag{12}$$

and

$$\tau^2 := \mathrm{Var}\left(Y_\bullet\right) = \frac{1}{R_\bullet^2}\left(\mathbf{W}_{PY}^T \cdot \mathbf{R}_{PY} \cdot \mathbf{W}_{PY}\right). \tag{13}$$

The no-correlation standard assumption means that $\mathbf{R}_{PY}$ is the identity matrix. Then (13) becomes

$$\tau^2 = \frac{1}{R_\bullet^2}\sum_{i=1}^{I} R_i^2\tau_i^2. \tag{14}$$

A.3 Modelling of the sum of normal claim amount CY and claim amount PY in the SST

Let

$$S_i = C_i^{CY,n} + \widetilde{R_i}, \tag{15}$$

$$Z_i = \frac{C_i^{CY,n} + \widetilde{R_i}}{\widetilde{P}_i + R_i} = \frac{\widetilde{P}_i X_i + R_i Y_i}{\widetilde{P}_i + R_i}. \tag{16}$$

Whereas $\widetilde{P}_i$ and R_i can be considered as the weights attached to X_i and Y_i respectively, the corresponding weight for Z_i is the sum

$$V_i = \widetilde{P}_i + R_i. \tag{17}$$

From (16) we immediately see that

$$\varphi_i^2 := \mathrm{Var}\left(Z_i\right) = \frac{\left(\widetilde{P}_i\sigma_i\right)^2 + 2\widetilde{P}_i\sigma_i R_i\tau_i \mathrm{Corr}\left(C_i^{CY}, C_i^{PY}\right) + \left(R_i\tau_i\right)^2}{V_i^2}. \tag{18}$$

At the end we are interested in the variance of the total claim amount CY and PY summed up over all lines of business. Denote by

$$\mathbf{R}_{CY,PY} = \mathrm{Corr}\left(\mathbf{C}^{CY}\ ,\ \mathbf{C}^{PY^T}\right)$$

the correlation matrix between the claim amounts CY and the claim amounts PY with entries

$$\mathbf{R}_{CY,PY}\left(i,j\right) = \mathrm{Corr}\left(C_i^{CY}, C_j^{PY}\right).$$

As correlation matrix for the joint random vector of claim amounts CY and claim amounts PY we obtain

$$\begin{aligned}\mathbf{R} &= \mathrm{Corr}\left(\begin{pmatrix}\mathbf{C}^{CY}\\ \mathbf{C}^{PY}\end{pmatrix}, \begin{pmatrix}\mathbf{C}^{CY}\\ \mathbf{C}^{PY}\end{pmatrix}^T\right)\\ &= \begin{pmatrix}\mathbf{R}_{CY} & \mathbf{R}_{CY,PY}\\ \mathbf{R}_{CY,PY} & \mathbf{R}_{PY}\end{pmatrix}.\end{aligned}$$

Let

$$S_\bullet = C_\bullet^{CY} + \widetilde{R}_\bullet, \tag{19}$$

$$Z_\bullet = \frac{C_\bullet^{CY} + \widetilde{R}_\bullet}{\widetilde{P}_\bullet + R_\bullet}. \tag{20}$$

Then we obtain

$$\mathrm{Var}\left(S_\bullet\right) = \begin{pmatrix}\mathbf{W}_{CY}\\ \mathbf{W}_{PY}\end{pmatrix}^T \cdot \mathbf{R} \cdot \begin{pmatrix}\mathbf{W}_{CY}\\ \mathbf{W}_{PY}\end{pmatrix}, \tag{21}$$

$$\mathrm{Var}\left(Z_\bullet\right) = \frac{1}{V_\bullet^2}\left\{\begin{pmatrix}\mathbf{W}_{CY}\\ \mathbf{W}_{PY}\end{pmatrix}^T \cdot \mathbf{R} \cdot \begin{pmatrix}\mathbf{W}_{CY}\\ \mathbf{W}_{PY}\end{pmatrix}\right\}, \tag{22}$$

where

$$V_\bullet = \widetilde{P}_\bullet + R_\bullet\,.$$

Since

$$Z_\bullet = \sum_{i=1}^{I}\frac{V_i}{V_\bullet}Z_i,$$

we can also express the variance of $Z_\bullet$ in terms of the Z-variables, i.e.

$$\begin{aligned}\mathrm{Var}\left(Z_\bullet\right) &= \sum_{i,j=1}^{I}\frac{V_iV_j\varphi_i\varphi_j}{V_\bullet^2}\mathrm{Corr}\left(Z_i, Z_j\right) \qquad (23)\\ &= \frac{1}{V_\bullet^2}\mathbf{V}^T(\mathbf{W}_{CY}^T \cdot \mathbf{R}_{CY} \cdot \mathbf{W}_{CY} + 2\cdot(\mathbf{W}_{CY}^T \cdot \mathbf{R}_{CY,PY} \cdot \mathbf{W}_{CY}) + \mathbf{W}_{PY}^T \cdot \mathbf{R}_{PY} \cdot \mathbf{W}_{PY})\mathbf{V},\end{aligned}$$

where $\mathbf{V} = (V_1, V_2, \ldots, V_I)^T$.

(22) is valid for any correlation matrix $\mathbf{R}$. The standard assumption in the current SST is that there is no correlation between current year risks and previous year risks, which means that $\mathbf{R}_{CY,PY}$ is a zero-matrix (matrix with only zeros). Together with the assumption that the reserve risks of different lob are not correlated we obtain

$$\mathrm{Var}\left(Z_\bullet\right) = \frac{\widetilde{P_\bullet}^2 \sigma^2 + R_\bullet^2 \tau^2}{\left(\widetilde{P_\bullet} + R_\bullet\right)^2}. \tag{24}$$

Discussion and remarks on the correlation assumption between CY and PY risks:

- The current standard assumption in the SST that CY risks and PY risks are not correlated, is questionable. There might well be *calendar year factors* affecting both, the CY and the PY results. The most obvious of these factors is claims inflation, which has an impact on both, CY claims and PY claims.

Model Assumptions A.3 (normal claim amount CY plus claim amount PY) *It is assumed that $S_\bullet$ has a lognormal distribution with*

$$\begin{aligned} E\left[S_\bullet\right] &= \widetilde{P_\bullet} + R_\bullet, \\ \mathrm{Var}\left(S_\bullet\right) &= \left(\mathbf{W}_{CY}, \mathbf{W}_{PY}\right)^T \cdot \mathbf{R} \cdot \left(\mathbf{W}_{CY}, \mathbf{W}_{PY}\right). \end{aligned}$$

Remark:

- This model assumption has to be interpreted in the way that a lognormal distribution is a sufficiently accurate approximation to the distribution of $S_\bullet$ for solvency purposes.

A.4 Modelling of the Big Claim Amount Current Year

Big claims are defined as single individual claims or claim events (natural catastrophes) with a claim amount exceeding a certain threshold (e.g. 1 m CHF or 5 m CHF). For both types of big claims it is assumed that the following model assumptions hold true.

Model Assumptions A.4 (big claim amount SST)

i) The big claim amount for line of business i has a compound Poisson distribution

$$C_i^{CY,b} = \sum_{\nu=1}^{N_i^b} Y_{i\nu}^b,$$

where the number of big claims N_i^b is Poisson-distributed with Poisson parameter λ_i^b and where the claim severities $Y_{i\nu}^b$, $\upsilon = 1, 2, ..., N_i^b$, are independent and independent from N_i^b with distribution function F_i.

ii) $\left\{C_i^{CY,b} : i = 1, 2, \ldots, I\right\}$ *are independent.*

Remarks:

- The claim severity distributions F_i are essentially assumed to be Pareto with Pareto parameters α_i. Upper limits can easily be taken into account by truncation of the Pareto distributions. Also xs-reinsurance is no problem to handle by this model.
- Big claim events like hail, wind-storm, flood etc. usually affect several lines of business. However, this is not relevant in this context. Such an event claim might then be associated to the lob with the highest expected claims load. The essential assumption is that all these big claims are independent and compound Poisson.

From these assumptions it follows that

$$C_\bullet^{CY,b} = \sum_{i=1}^{I} C_i^{CY,b}$$

is again compound Poisson with parameter

$$\lambda = \lambda_\bullet^b = \sum_{i=1}^{I} \lambda_i^b$$

and claim severity distribution

$$F = \sum_{i=1}^{n} \frac{\lambda_i^b}{\lambda_\bullet^b} F_i.$$

The compound Poisson-distribution of $C_\bullet^{CY,b}$ can for instance be calculated by means of the Panjer-algorithm.

A.5 Aggregation

Let

$$\widetilde{T} = C_\bullet^{CY,n} + C_\bullet^{CY,b} + C_\bullet^{PY}$$

be the total claim amount before scenarios. According to model-assumptions A.3, the distribution of $S_\bullet = C_\bullet^{CY} + C_\bullet^{PY}$ is lognormal and according to model-assumptions A.4, the distribution of $C_\bullet^{CY,b}$ is compound Poisson. Thus the distribution function $\widetilde{F}(x)$ of $\widetilde{T}$ is obtained by convoluting the lognormal distribution of $S_\bullet$ with the compound Poisson distribution of $C_\bullet^{CY,b}$.

A.6 Taking the scenarios into account

Insurance scenarios SC_k^{ins} are situations or events producing an estimated loss c_k (face amount) with probability p_k. In the SST standard model, beside prescribed scenarios, the company should also think about company specific scenarios. Hence the number K of the scenarios considered by a company is not fixed. However, in the standard SST it is assumed that the scenarios $SC_k^{ins}, k = 1, 2, \ldots, K$ exclude each other, i.e. that only one of the scenarios can materialize during the next year. This exclusion property was made because of simplicity reasons and is not such a big restriction as it might seem on the first side. If one wants to incorporate the situation, that e.g. SC_i^{ins} and SC_j^{ins} might occur in the same year, one can add a new scenario SC_{new}^{ins} with face amount $c_{new} = c_i + c_j$ and a corresponding probability p_{new}, for instance $p_{new} = p_i p_j$ if the the two scenarios are assumed to be independent.

Let

$$T = C_{\bullet}^{CY,n} + C_{\bullet}^{CY,b} + C_{\bullet}^{PY} + SC_{\bullet}^{ins},$$

be the total claim amount including scenarios, and denote by $F(x)$ the distribution of T. Define

$$\begin{aligned} p_0 &= 1 - \sum_{k=1}^{K} p_k \text{ and} \\ c_0 &= 0. \end{aligned}$$

Then

$$F(x) = \sum_{k=0}^{K} p_k \widetilde{F}(x - c_k),$$

where $\widetilde{F}(.)$ is the distribution before scenarios.

B Parameter Estimation

The standard parameter values in the STT and in solvency II differ substantially. This fact by itself reveals that the estimation of the parameters is a problem on its own.

The SST framework does not yet provide formulas for estimating the parameters from individual data. In the following we present such estimators.

B.1 Estimation of the random fluctuation risk normal claims CY

In Section A.1 we have seen that

$$\sigma_{i,fluct}^2 = CoVa^2\left(Y_i^{(\upsilon)}\right) + 1.$$

Hence we have to estimate the coefficient of variation of the claim sizes. For this purpose we calculate first the observed coefficient of variations for different accident years j, that is

$$\widehat{CoVa}_{ij} = \sqrt{\frac{\frac{1}{N_{ij}-1}\sum_{\upsilon=1}^{N_{ij}}\left(Y_{ij}^{(\nu)} - \overline{Y}_i\right)^2}{\overline{Y}_i^2}}, \tag{25}$$

where N_{ij} is the number of normal claims of lob i in accident year j, $Y_{ij}^{(\nu)}, \upsilon = 1, 2, \ldots, N_{ij}$, the individual claim amounts and $\overline{Y}_i$ the mean of the $Y_{ij}^{(\nu)}$. However, one should have in mind that the individual claim amounts $Y_{ij}^{(\nu)}$ in recent accident years j consist to a great part of case estimates. It is a known fact that, especially in long tail lines of business, (25) underestimates in recent accident years the true ultimate coefficient of variation. Therefore, the values resulting from (25) should first be extrapolated to their ultimate value. The following table shows the development triangle of the coefficient of variation in motor liability of a big Swiss insurance company.

Development triangle CoVa claim amounts in motor liability

AY/DY	0	1	2	3	4	5	6	7	8	9
1998	6.1	7.1	8.1	8.5	9.1	9.7	10.1	10.2	10.2	10.9
1999	6.1	7.6	8.1	9.0	10.0	10.3	10.6	10.6	11.2	11.2
2000	5.8	6.9	8.4	8.9	9.4	9.5	9.5	10.3	10.3	10.3
2001	5.8	7.4	8.9	9.5	9.6	9.6	10.4	10.7	10.7	10.7
2002	5.7	7.8	9.0	9.0	9.2	9.9	10.3	10.6	10.6	10.6
2003	6.6	8.5	8.8	9.1	9.8	10.2	10.6	10.9	10.9	10.9
2004	6.6	8.5	9.2	10.1	10.7	11.1	11.5	11.9	11.9	11.9
2005	5.2	7.2	8.7	9.2	9.7	10.1	10.5	10.8	10.8	10.8
2006	5.4	7.5	8.5	9.0	9.5	9.9	10.3	10.6	10.6	10.6
2007	5.7	7.4	8.4	8.8	9.4	9.7	10.1	10.4	10.4	10.4
									mean	10.8

The values above the diagonals are the observed empirical coefficients of variation resulting from (25) of accident years i calculated at different development years. The values below the diagonals are simple chain ladder forecasts. We can see from this table that for instance in the development years 0 or 1 the coefficients of variation are substantially underestimated compared to the ultimate values. We can also see that the observed coefficients of variation are rather stable and that the mean of the extrapolated coefficients of variation (column 9) over the different accident years is an accurate estimator for the coefficient of variation of the claim sizes. Indeed, this parameter can usually be estimated with great accuracy from the own data.

B.2 Estimation of the parameter risk of the normal claim amount CY

For the parameter risk, most companies in Switzerland use the default standard values provided by the supervision authority. One of the reasons is that there has not yet been developed an estimator for the parameter risk for the own data which is based on a sound actuarial basis. In the following we try to fill this gap and we will present such estimators.

We assume that we have observed historical data on the accident years $j = 1, 2, \ldots, J$. We consider a specific lob i. To simplify notation we drop in the following the index i and we write for instance X_j for the observation of the specific lob considered in the year j. It is assumed,

i) that each year is characterised by its characteristics $\mathbf{\Theta}_j = (\Theta_{j1}, \Theta_{j2})^T$ and that for each year the model-assumptions A.1 are fulfilled with underlying claim frequency parameter λ_j and claim severity parameter μ_j,

ii) that the coefficient of variation for the claim sizes $CoVa\left(Y_j^{(\nu)}\right)$ is the same for all years,

iii) that random variables belonging to different years are independent and $\mathbf{\Theta}_1, \mathbf{\Theta}_2, \ldots, \mathbf{\Theta}_J$ are independent and identically distributed.

From the above assumptions it follows that the random variables X_j are independent with

$$\operatorname{Var}(X_j) = \sigma^2_{param} + \frac{\sigma^2_{fluct}}{\nu_j} \tag{26}$$

$$\simeq \sigma^2_{param} + \frac{\widetilde{\sigma}^2_{fluct}}{\widetilde{P}_j}, \tag{27}$$

where

$$\begin{aligned} \nu_j &= \text{number of a priori expected claims}, \\ \widetilde{P}_j &= E\left[C_j^{CY,n}\right], \\ \sigma^2_{fluct} &= CoVa^2\left(Y^{(\upsilon)}\right) + 1, \end{aligned}$$

and with

$$\begin{aligned} E\left[X \mid \mathbf{\Theta}\right] &= \Theta_1 \Theta_2, \\ \operatorname{Var}\left(X \mid \mathbf{\Theta}\right) &= \frac{1}{w\lambda} \cdot \Theta_1 \Theta_2^2 \left(CoVa^2\left(Y\right) + 1\right). \end{aligned}$$

Hence the observations X_j, $j = 1, 2, \ldots, J$, fulfil the conditions of the Bühlmann-Straub model (see for instance [1], Chapter 4). However, we do not have the "standard situation" of the Bühlmann Straub model, where we have a collective of risks $i = 1, 2, \ldots, I$, and where for each of these risks observations $X_{ij}, j = 1, 2, \ldots, n$, over several years are available. Here the risks are the different years $j = 1, 2, \ldots, J$ and for each of these "risks", one has only one observation X_j. In the standard situation of the Bühlmann-Straub model, the observations over several years for each risk are used to estimate the within risk variance component. But here the within risk variance is σ^2_{fluct}, which can be estimated otherwise as described in Section B.1.

Therefore we can use the standard estimators in the Bühlmann-Straub model (see e.g. [1], Chapter 4.8) and obtain

$$\widehat{\sigma}^2_{param} = c \cdot \left\{ \frac{J}{J-1} \sum_{j=1}^{J} \frac{w_j}{w_\bullet} \left(X_j - \overline{X}\right)^2 - \frac{J\,\widehat{\sigma}^2_{fluct}}{n_\bullet} \right\}, \tag{28}$$

where

$$\begin{aligned} X_j &= \frac{C_j^{CY,n}}{\widetilde{P}_j}, \\ C_j^{CY,n} &= \text{normal claim amount CY of year } j, \\ w_j &= \widetilde{P}_j = \text{ pure risk premium in year } j, \end{aligned}$$

$$
\begin{aligned}
c &= \frac{J-1}{J}\left\{\sum_{j=1}^{J}\frac{w_j}{w_\bullet}\left(1-\frac{w_j}{w_\bullet}\right)\right\}^{-1}, \\
\widehat{\sigma}^2_{fluct} &= \widehat{CoVa}^2\left(Y^{(v)}\right) \text{ estimator of the coefficient of} \\
&\quad \text{variation of the claim sizes,} \\
n_\bullet &= \text{observed number of claims.}
\end{aligned}
$$

Whereas the total claim amounts $C_j^{CY,n}$ are known figures and recorded in the files of a company, this is not the case for the corresponding pure risk premiums $\widetilde{P}_j = E\left[C_j^{CY,n}\right]$. Hence, before being able to apply (28) one has to determine $\widetilde{P}_j$, which are used for calculating the X_j.

Denote by

$$LR_j = \frac{C_j^{CY,n}}{P_j}$$

the observed claims ratio for normal claims in year j, where P_j is the earned premium. Under the assumption that

$$E\left[LR_j\right] = \mu_{LR} \tag{29}$$

is the same for all years, it is suggested to use

$$\widetilde{P}_j = \overline{LR}\cdot P_j$$

where

$$\overline{LR} = \sum_{j=1}^{J}\frac{P_j}{P_{i\bullet}}LR_j.$$

Often (29) is not fulfilled because of things like business cycles and premium policies. Then the premiums have first to be adjusted by such effects, such that the loss ratios calculated with the adjusted "on level" premiums fulfils (29).

Finally, we have to decide which weights w_j should be used in (28).We suggest to take the $\widetilde{P}_j$ as weights, which is justified by the variance property (27).

The estimator (28) is based on the properties and the variance structure of X_j. It does, however not make use of the fact that according to (5)

$$\sigma^2_{param} \simeq \mathrm{Var}\left(\Theta_1\right) + \mathrm{Var}\left(\Theta_2\right). \tag{30}$$

An alternative approach to (28) is therefore to estimate $\mathrm{Var}(\Theta_1)$ and $\mathrm{Var}(\Theta_2)$ directly, which then involves the claim frequencies and the claim averages. Hence the following estimators are recommended for lines of business where there is a "natural" volume measure, which is necessary that the claim frequencies are comparable over time. This is the case for personal lines like motor liability, household insurance etc., but not for corporate property, corporate liability or industrial fire insurance.

Let

$$
\begin{aligned}
F_j &= \frac{N_j}{\nu_j}, \\
\text{where } N_j &= \text{number of claims in year } j, \\
\nu_j &= \text{a priori expected number of claims in year } j.
\end{aligned}
$$

Note that F_j is a "standardised" frequency with $E[F_j] = 1$. If there are no trends in the non-standardised claim frequency, that is if

$$E\left[\frac{N_j}{JR_j}\right] = \lambda \text{ for all } j, \tag{31}$$

then one can put

$$\begin{aligned} \nu_j &= \widehat{\lambda} \cdot JR_j, \\ \text{where } JR_j &= \text{ number of risks in year } j, \\ \widehat{\lambda} &= \frac{N_\bullet}{JR_\bullet}. \end{aligned}$$

If there are trends, then use $\nu_j = \widehat{\lambda}_j \cdot JR_j$, where $\widehat{\lambda}_j$ is the trend adjusted estimate of the a priori claim frequency in year j.

Under model-assumptions A.1, where N_j is assumed to be conditionally Poisson, it holds that

$$\mathrm{Var}(F_j) = \mathrm{Var}(\Theta_1) + \frac{\lambda}{\nu_j}$$

and one can show that

$$\widehat{\sigma^2_{\Theta_1}} = \left(c \cdot \frac{\nu_\bullet}{J}\right)^{-1} \left(\frac{V_F}{\overline{F}} - 1\right) \tag{32}$$

where

$$\begin{aligned} V_F &= \frac{1}{J-1} \sum_{j=1}^{J} \nu_j \left(F_j - \overline{F}\right)^2, \\ \overline{F} &= \sum_{j=1}^{J} \frac{\nu_j}{\nu_\bullet} F_j, \\ c &= \sum_{j=1}^{J} \frac{\nu_j}{\nu_\bullet} \left(1 - \frac{\nu_j}{\nu_\bullet}\right). \end{aligned}$$

is an unbiased estimator of $\mathrm{Var}(\Theta_1)$.

For estimating $\mathrm{Var}(\Theta_2)$ one has to look at the observed claims averages

$$\overline{Y_j} = \frac{1}{N_j} \sum_{\upsilon=1}^{N_j} Y_j^{(\nu)}$$

in different years. Because of inflation or possible other trends, one has first to adjust the claim sizes in a given year j by these factors to bring them on the same level, which can be done for instance by linear regression. After these adjustments, the claim sizes and claim averages fulfil the Bühlman-Straub credibility model for claim sizes as presented in Chapter 4.11 of [1] and one can use this theory to estimate $\mathrm{Var}(\Theta_2)$.

B.3 Estimation of the Pareto Parameters for Big Claims

We consider again a specific lob i and drop the index i to simplify notation. Assume that $Y_1, Y_2, ...$ are the big claims above a certain threshold c and we assume that they are Pareto-distributed with Pareto-parameter ϑ.

Assume you have observed n such claims. It is well known that

$$\widehat{\vartheta} = \left(\frac{1}{n-1} \sum_{\nu=1}^{n} \ln \left(\frac{Y_\nu^b}{c} \right) \right)^{-1} \tag{33}$$

is an unbiased estimator of ϑ with

$$\begin{aligned} E\left[\widehat{\vartheta}\right] &= \vartheta, \\ CoVa\left(\widehat{\vartheta}\right) &= \frac{1}{\sqrt{n-2}}. \end{aligned} \tag{34}$$

The number of observed big claims of an individual company in the considered lob might be rather small. From (34) it is seen that the uncertainty of this estimate then becomes fairly big. Hence, it is desirable to take also into account in some way the standard value of the SST, which can be considered as an estimate gained from industry-wide data.

Indeed, the estimator $\widehat{\vartheta}$ fulfils the Bühlmann-Straub credibility model and we can use credibility techniques to combine the individual experience with the a priori estimate given by the standard value of the SST. The credibility estimator is given by (see [11] or [1], Chapter 4.14)

$$\widehat{\vartheta}^{cred} = \alpha \widehat{\vartheta} + (1-\alpha)\vartheta_0 \tag{35}$$

where $\widehat{\vartheta}$ is as in (33) and where

$$\begin{aligned} \alpha &= \frac{n-2}{n-1+\kappa}, \\ \vartheta_0 &= \text{standard value from the SST}, \\ \kappa &= CoVa\left(\Theta\right)^{-2}. \end{aligned}$$

Here $CoVa(\Theta)$ denotes the coefficient of variation of the Pareto parameter within the different companies. It could be estimated for instance by the supervision authority from the data of the different companies, or one can assess it in a pure Bayesian way, let's say to assume that it is 25%, which would result in a value of 16 for κ. For instance if your individual estimator is based on 10 observed big claims, then you would give a credibility weight of 32% to your own Pareto-estimate and if your estimator is based on 30 observed big claims, then the credibility weight given to your own estimate would be 62%.

B.4 Estimation of the Reserve Risk

We believe that the reserve risk should be determined based on the techniques used for estimating these reserves. The most known formula for estimating the variance or mean squared error of the reserves is the famous formula of Mack for the chain ladder reserving method (see [9]). However, Mack's formula measures the ultimate reserve risk whereas for solvency purposes we need the one-year reserve risk.

The one-year reserve risk is best understood when we look at the recursive formula for the chain ladder reserving method. This has been done in [2]. The formula derived there coincides with the result found in [10].

In the following, we briefly summarize these results.

Assume that at time I, there is given a development triangle or trapezoid

$$\mathcal{D}_I = \{C_{i,j} : 0 \leq i \leq I,\ 0 \leq j \leq J,\ i + j \leq I\},$$

where $C_{i,j}$ denote cumulative claims payments or cumulative incurred claims of accident year i in development year j and where $C_{i,J}$ is the ultimate claim. Let

$$\begin{aligned} \widehat{C}_{i,J} &= C_{i,I-i} \prod_{j=I-i}^{J-1} \widehat{f}_j, \\ \widehat{R}_i &= \widehat{C}_{i,J} - C_{i,I-i}^{paid}, \\ \widehat{\sigma}_j^2 &= \frac{1}{I-j-1} \sum_{i=0}^{I-j-1} C_{i,j} \left(F_{i,j} - \widehat{f}_j \right)^2. \end{aligned} \tag{36}$$

$$\begin{aligned} F_{ij} &= \frac{C_{i,j+1}}{C_{i,j}}, \\ S_j^{[k]} &= \sum_{i=0}^{k} C_{i,j}, \\ \widehat{f}_j &= \frac{S_{j+1}^{[I-j-1]}}{S_j^{[I-j-1]}}, \end{aligned} \tag{37}$$

Note that the $\widehat{f}_j$ are the well known chain ladder factors, $\widehat{C}_{i,J}$ is the chain ladder forecast for the ultimate claim and $\widehat{R}_i$ is the chain ladder reserve for accident year i.

Then in [5] and [10] the following result has been derived for the one-year reserve risk.

Theorem B.1 (one-year mse) *The one-year mean square error can be estimated as follows:*

i) single accident year i

$$mse\left(\widehat{R}_i\right) = C_{i,I-i}\Gamma_{I-i} + C_{i,I-i}^2 \Phi_{I-i} \tag{38}$$

where

$$\begin{aligned} \Gamma_{I-i} &= \widehat{\sigma}_{I-i}^2 \left(1 + \frac{C_{i,I-i}}{S_{I-i}^{[i]}} \right) \prod_{j=I-i+1}^{J-1} \widehat{f}_j, \\ \Phi_{I-i} &= \frac{\widehat{\sigma}_{I-i}^2}{S_{I-i}^{[i]}} \prod_{j=I-i+1}^{J-1} \widehat{f}_j^{\,2}, \end{aligned}$$

ii) all accident years

$$mse\left(\widehat{R}_\bullet\right) = \sum_{i=I-J+1}^{I} mse\left(\widehat{R}_i\right) + 2\sum_{I-J+1\le i<k\le I}^{I} C_{i,I-i}\widehat{C_{k,I-i}}\left(\Gamma_{I-i}+\Phi_{I-i}\right), \qquad (39)$$

where $\widehat{C_{k,I-i}}$ *is the chain ladder forecast of* $C_{k,I-i}$*.*

Remarks:

- Since the reserves $\widehat{R}_i$ are proportional to $C_{i,I-i}$, model assumptions A.1 (or (11), respectively) are consistent with model assumptions A.2 (or (38) respectively). In (38) the term $C_{i,I-i}\Gamma_{I-i}$ corresponds to the random fluctuation risk and the term $C^2_{i,I-i}\Phi_{I-i}$ to the parameter risk.
- We believe that the parameter risk for the chain ladder reserving method can be treated in a rigorous mathematical way only in a Bayesian set-up (see [2], [**?**], [5]). With the Bayesian set-up and taking a non-informative prior the resulting estimates are slightly different from Mack's formula for the ultimate risk or the above formula for the one-year risk. Mack's formula and the above formula are then obtained by a first order Taylor approximation from the corresponding results with the Bayesian approach.

When looking at the reserve risk from a solvency point of view, we are interested in the 100- or 200-year adverse event. What are such events that first come into our mind? There are things like a high inflation or a change in legislation like a decrease of the technical interest rate to calculate the lump sums in motor liability insurance, hence situations with a high adverse calendar year effect. But such events are usually not observed in the triangles and not captured by chain-ladder or similar reserving methods. Hence they are also not reflected by the above estimator of the reserve risk. Thus the question arises whether we do the right thing when looking at the reserve risk from a solvency point of view. The answer is that the above method is adequate for the reserve risk except in extraordinary situations with a huge adverse calendar year effect. Hence, it seems absolutely necessary to complement the reserve risk calculation with a scenario of a hyper-inflation or another adverse situation which might happen. Indeed, nobody knows whether the actual finance crisis will not lead to high inflation in some years.

The following example is taken from [6]. First you find below a development triangle in private-liability of a major Swiss insurance company. For confidentiality reasons the figures are multiplied by a constant factor. The figures above the diagonal are the observed cumulative payments, whereas the figures below the diagonal show the chain ladder forecasts.

development triangle cumulative payments

acc. year \ dev. year	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	Ultimate
1979	3'670	5'817	6'462	6'671	6'775	7'002	7'034	7'096	7'204	7'326	7'395	7'438	7'468	7'541	7'580	7'677	7'794	7'919	8'047	8'051	8'152	8'152
1980	4'827	7'600	8'274	8'412	9'059	9'197	9'230	9'277	9'331	9'435	9'864	9'621	9'654	9'704	9'743	9'745	9'745	9'748	9'749	9'749	9'749	9'749
1981	7'130	10'852	11'422	12'024	12'320	12'346	12'379	12'426	12'460	12'500	12'508	12'547	12'557	12'624	12'639	12'644	12'647	12'666	12'711	12'713	13'063	13'063
1982	9'244	13'340	13'758	13'853	13'894	13'942	13'980	14'001	14'012	14'070	14'085	14'098	14'111	14'118	14'120	14'133	14'141	14'145	14'166	14'167	14'167	14'167
1983	10'019	14'223	15'403	15'579	15'732	15'921	16'187	16'420	16'531	16'559	16'568	16'585	16'597	16'614	16'617	16'619	16'681	16'681	16'687	16'711	16'748	16'748
1984	9'966	14'599	15'181	15'431	15'506	15'538	15'906	16'014	16'537	16'833	16'951	17'038	17'040	17'195	17'298	17'307	17'308	17'308	17'309	17'310	17'311	17'311
1985	10'441	15'043	15'577	15'784	15'926	16'054	16'087	16'107	16'311	16'366	16'396	16'414	16'419	16'426	16'480	16'480	16'480	16'480	16'482	16'482	16'482	16'482
1986	10'578	15'657	16'352	16'714	17'048	17'289	17'632	17'646	17'662	17'678	17'693	17'700	17'706	17'706	17'712	17'713	17'718	17'719	17'719	17'719	17'719	17'719
1987	11'214	16'482	17'197	17'518	18'345	18'480	18'505	18'520	18'578	18'633	18'671	18'689	18'746	18'772	18'774	18'804	18'804	18'804	18'806	18'806	18'809	18'809
1988	11'442	17'621	18'465	18'693	18'882	18'965	20'464	20'522	20'558	20'607	20'611	21'250	21'257	22'505	22'509	22'516	22'518	22'522	22'523	22'527	22'611	22'611
1989	11'720	17'779	18'655	18'940	19'098	19'368	19'970	20'162	20'195	20'415	20'510	20'594	20'657	20'752	20'823	20'899	20'983	21'124	21'622	21'627	21'708	21'708
1990	13'293	20'689	21'696	22'439	22'798	23'054	23'394	23'554	23'888	24'061	24'096	24'301	24'356	24'389	24'391	24'784	24'784	24'794	24'894	24'900	24'993	24'993
1991	15'063	22'917	23'543	24'032	24'156	24'232	24'360	24'410	24'884	24'896	24'968	25'031	25'172	25'459	25'460	25'470	25'734	25'774	25'877	25'883	25'980	25'980
1992	16'986	23'958	25'090	25'392	25'546	26'099	26'129	26'149	26'166	26'231	26'328	26'743	27'023	27'048	27'075	27'168	27'234	27'276	27'386	27'392	27'494	27'494
1993	16'681	24'867	25'871	26'463	26'941	27'120	27'164	27'183	27'250	27'490	27'497	27'561	27'565	27'582	27'706	27'787	27'855	27'898	28'010	28'017	28'121	28'121
1994	17'595	25'152	26'140	27'090	27'568	27'637	27'805	28'003	28'223	28'240	28'245	28'250	28'257	28'297	28'347	28'430	28'499	28'543	28'658	28'665	28'772	28'772
1995	16'547	25'396	26'506	27'043	27'866	27'882	27'903	27'933	28'492	28'545	28'564	28'686	28'695	28'897	28'948	29'033	29'104	29'149	29'266	29'273	29'382	29'382
1996	15'449	22'702	23'909	24'690	26'083	26'525	26'567	26'640	26'695	26'801	26'814	27'018	27'078	27'269	27'317	27'397	27'463	27'506	27'616	27'623	27'726	27'726
1997	18'043	26'918	28'256	28'569	28'964	29'268	29'344	29'393	30'159	30'936	30'966	31'122	31'190	31'410	31'466	31'558	31'635	31'684	31'811	31'819	31'937	31'937
1998	17'655	26'241	27'369	28'063	28'346	28'780	29'024	29'180	29'250	29'575	29'660	29'809	29'875	30'085	30'138	30'227	30'300	30'347	30'469	30'476	30'590	30'590
1999	16'789	25'547	27'099	27'801	27'919	28'052	30'020	30'035	30'456	30'663	30'750	30'905	30'973	31'192	31'247	31'338	31'415	31'463	31'590	31'597	31'715	31'715
2000	15'538	23'830	25'202	26'462	27'056	27'480	27'564	27'612	27'873	28'062	28'143	28'284	28'347	28'546	28'597	28'681	28'751	28'795	28'911	28'917	29'025	29'025
2001	15'113	23'405	26'822	27'711	28'080	29'203	29'647	29'751	30'032	30'236	30'323	30'475	30'542	30'758	30'812	30'903	30'978	31'025	31'150	31'157	31'274	31'274
2002	14'543	22'674	23'603	24'159	24'242	24'425	24'767	24'854	25'089	25'260	25'332	25'459	25'515	25'695	25'741	25'816	25'879	25'919	26'023	26'029	26'126	26'126
2003	14'590	22'337	23'442	24'031	24'665	24'942	25'292	25'380	25'621	25'794	25'868	25'998	26'056	26'239	26'286	26'363	26'427	26'468	26'574	26'580	26'680	26'680
2004	13'976	21'527	22'615	23'242	23'653	23'918	24'254	24'339	24'569	24'736	24'806	24'931	24'986	25'162	25'207	25'281	25'342	25'381	25'483	25'489	25'585	25'585
2005	12'932	20'118	21'309	21'824	22'209	22'459	22'774	22'853	23'070	23'226	23'293	23'410	23'462	23'627	23'669	23'738	23'796	23'832	23'928	23'934	24'023	24'023
2006	12'538	20'357	21'435	21'953	22'341	22'592	22'909	22'989	23'206	23'364	23'431	23'548	23'600	23'767	23'809	23'879	23'937	23'974	24'070	24'076	24'166	24'166
2007	12'888	19'413	20'441	20'935	21'304	21'544	21'846	21'922	22'130	22'280	22'344	22'456	22'506	22'664	22'704	22'771	22'826	22'861	22'953	22'959	23'045	23'045
																						673'153

The next table shows the resulting chain ladder reserves and the corresponding standard-deviation for each accident year and for the total. The ultimate reserve risk calculated with Mack's formula amounts to 13.3%, whereas the one year reserve risk according to formula (39) reduces to 6.6%.

		ultimate (Mack)		one year	
acc. year	Cl-Reserve	CoVa	sqrt mse	Vko_CDRi	sqrt mse
1979-1987	-	-	-	-	-
1988	84	206.4%	174	206.4%	174
1989	86	198.0%	170	30.3%	26
1990	199	129.2%	257	91.0%	181
1991	246	111.7%	275	36.7%	90
1992	326	92.1%	301	32.7%	107
1993	415	78.9%	328	31.0%	129
1994	475	70.8%	336	14.4%	68
1995	687	73.6%	506	54.9%	377
1996	708	70.0%	495	12.9%	91
1997	971	60.6%	589	26.0%	252
1998	1'015	59.7%	605	19.7%	200
1999	1'259	51.4%	647	16.3%	205
2000	1'413	46.8%	662	17.6%	249
2001	1'627	42.8%	696	7.1%	116
2002	1'701	48.4%	823	31.3%	532
2003	2'015	43.0%	867	12.4%	249
2004	2'343	41.2%	964	16.3%	381
2005	2'714	34.4%	933	9.7%	262
2006	3'809	28.6%	1'088	14.7%	558
2007	10'157	13.1%	1'328	7.9%	801
Total	32'250	13.3%	4'294	6.6%	2'131

Finally, we should consider an adverse extraordinary scenario and also take it into account when considering the reserve risk. If we assume that claims inflation increases by

3 %-points (additional inflation to the one already existent in the development triangle) and stays on this level for 10 years, then this would create a reserve loss of 4'401 or 13.6% of the reserves. Inflation scenarios would also affect the reserves of other lob, and its impact should be taken into account as a reserve scenario for the total business of a company.

The observations in the development triangles might vary quite a lot due to random fluctuations, in particular for small and medium sized companies. It would therefore be helpful to know an industry wide payment pattern and a technique how to combine the individual pattern with the industry wide pattern to obtain an optimal estimate. Here we just want to mention that such a technique was presented in [5]. The main idea is to use credibility to obtain credibility estimates of the chain ladder factors, which are then a weighted mean between the industry wide factors and the factors obtained from the company's own triangle.

B.5 On the Estimation of the Correlation Matrices

We believe that we should first think about the reasons why risks of different lob or the premium and reserve-risk of the same lob are correlated. The main reasons for such correlations are calendar year effects affecting the different risks simultaneously.

To understand the impact of diagonal factors let us have a closer look at the reserve risk. However, the same considerations could also be applied to the CY-risk.

Using the same notation as in Section A.2 we find

$$\widetilde{R_i} = Y_i R_i \tag{40}$$

with

$$\begin{aligned} E\left[Y_i\right] &= 1, \\ \operatorname{Var}\left(Y_i\right) &= \tau_i^2. \end{aligned}$$

Assume that (40) reflects the situation before calendar year effect and that the reserves risk of different lob are independent, that is

$$Cov\left(Y_i, Y_j\right) = 0 \quad \text{for } i \neq j.$$

Add now a diagonal effect Δ independent of $\widetilde{R_i}$ with $E\left[\Delta\right] = 1$ affecting different lob simultaneously (e.g. claims inflation) and denote by

$$\begin{aligned} \widetilde{R_i}^* &= Y_i R_i \Delta, \\ C_i^{PY*} &= \widetilde{R_i}^* - R_i, \end{aligned}$$

the a posteriori best estimate of the outstanding liabilities L_i, respectively the claim amount PY after calendar year effect. Then

$$\begin{aligned} \operatorname{Var}\left(\widetilde{R_i}^*\right) &= R_i^2 \left(E\left[Y_i^2\right] E\left[\Delta^2\right] - 1\right) \\ &= R_i^2 \left\{\left(E\left[Y_i^2\right] - 1\right)\left(1 + E\left[\Delta^2\right] - 1\right) + E\left[\Delta^2\right] - 1\right\} \\ &= R_i^2 \left\{\tau_i^2 + \sigma_\Delta^2 + \tau_i^2 \sigma_\Delta^2\right\} \\ &\simeq R_i^2 \left\{\tau_i^2 + \sigma_\Delta^2\right\}, \end{aligned}$$

where in the last equation we have assumed that $\tau_i^2\sigma_\Delta^2 << \tau_i^2 + \sigma_\Delta^2$ and where

$$\sigma_\Delta^2 = \mathrm{Var}\left(\Delta\right).$$

Under the assumption that the calendar year effect is also effective for lob j we obtain

$$\begin{aligned} Cov\left(\widetilde{R}_i^*, \widetilde{R}_j^*\right) &= Cov\left(C_i^{PY^*}, C_j^{PY^*}\right) \\ &= R_i R_j \mathrm{Var}\left(\Delta\right), \\ \mathrm{Corr}\left(\widetilde{R}_i^*, \widetilde{R}_j^*\right) &= \mathrm{Corr}\left(C_i^{PY^*}, C_j^{PY^*}\right) \\ &= \frac{\sigma_\Delta^2}{\sqrt{\tau_i^2+\sigma_\Delta^2}\sqrt{\tau_j^2+\sigma_\Delta^2}}. \end{aligned} \tag{41}$$

(41) is an intuitive and handy formula, which gives quite a good qualitative insight: correlation induced by a calendar year effect becomes smaller the smaller σ_Δ^2 is compared to τ_i^2 and τ_j^2. As an example consider claims inflation. Assume that the yearly standard deviation for claims inflation is 1% and that the reserve risks before the calendar year effect "inflation" for lob i and j are 3%. Then we obtain from (41) that

$$\mathrm{Corr}\left(C_i^{PY^*}, C_j^{PY^*}\right) = 11\%.$$

If we increase the calendar year inflation to 3%, the correlation would be 50%. This shows for instance, that the correlation induced by varying inflation is bigger in countries with high inflation than in countries with low inflation. The same considerations can also be applied to other calendar year effects and might help experts to assess the correlation matrices.

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