How sensitive is our exposure to inflation?

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On the role of inflation for (re) insurers

• BUSINESS Inflation erodes the underwriting discipline and profitability
  
  − Pricing
    
    − ‘Calculate the premium required to cover all expect cost components!’
      
      Inflation increases the required premium, particularly for long-tailed business.
    
    − ‘The insurer receives a fixed premium today but has to compensate for the value of a claim in some future!’ A corresponding inflation caused gap would increase the price.
  
  − Demand for insurance
    
    − ‘Buying insurance is forward-looking behavior!’ In high-inflation regimes due to higher uncertainty, less mid-term and long-term investments are made.
      
      In the 1970s, higher inflation velocity coincides with subsequent drops in underwriting and profitability.
  
• RISK Unexpected inflation, deviation from the expected, is a risk for reserving.
Sensitivity: towards its definition in a One-Year-view

Given that at the end of the year inflation deviates from what is expected by n base point, how much does actual payment then deviate from the expected?

\[ \delta j := \frac{j - \hat{j}}{\hat{j}} \]

<table>
<thead>
<tr>
<th>expected inflation</th>
<th>( j )</th>
<th>( \hat{j} )</th>
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<tbody>
<tr>
<td>expected payment</td>
<td>( \hat{z} )</td>
<td>( z )</td>
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\[ \delta z := \frac{z - \hat{z}}{\hat{z}} \]
Sensitivity: its definition as an elasticity

**Definition [Sensitivity]** The sensitivity of an exposure is defined as the elasticity of the payment function with respect to economic inflation, i.e.

\[
R := \frac{\delta z}{\delta j} \bigg| \delta j = 0
\]

Note that according to this definition, \( z = \hat{z}(1 + R \delta j) \) so that \( R \delta j \) might be regarded as the rate of claims inflation.
How to estimate the sensitivity of a contract from real data

Varying the inflation level is numerically equivalent to varying the deductible.

\[ Z = \max(jX - d, 0) \]

\[ = j \max \left( X - \frac{d}{j}, 0 \right) \]

\[ R := \left. \frac{\delta Z}{\delta j} \right|_{\delta j = 0} \]
Distribution of sensitivities in a Line of Business

mean sensitivity: 1.80
median sensitivity: 1.58
MAD sensitivity: 0.20

The sensitivity of the portfolio is the convex sum of the individual sensitivities of the contracts involved:

\[ \mathcal{R} = \sum_i \zeta_i \mathcal{R}^{(i)} \]

\[ \zeta_i = \frac{z_i(0)}{\sum_j z_j(0)} \]
‘Experienced grown’ rule of thumb values and statistical estimate

RoTh:
range $[1, 2]$
mean 1.8
median $1.8 \pm 0.3$

RoTH:
range $[1.5, 3]$
mean 2.5
median $2.0 \pm 0.4$
Back to the theoretical roots: What do these numbers mean

Let losses be distributed according to some distribution $\varphi$, whose first moment exists. Then the sensitivity of an Excess-of-Loss contract with retention $r \geq 0$ and infinite cover is

$$R^{\infty}_r(\varphi) = 1 + \frac{r}{\mathbb{E}[(Y-r) | Y \geq r]}$$

Let claim sizes $C$ be distributed according to some $\varphi$, whose first moment exists. According to eqn (1) we have that $j = \hat{j}(1 + \delta j)$, where $|\delta j| < 1$ is sufficiently small. Further let $X = jC$ and $Y = \hat{j}C$, so that $X = (1 + \delta j)Y$. Expected payment is

$$\mathbb{E}(X - r)^+ = z(\delta j) =: (1 + \delta j)\mathbb{E} \left( Y - \frac{r}{1+\delta j} \right)^+$$

Expanding the expected payment in $\delta j = 0$ yields

$$z(\delta j) = z(0) + z'(0)\delta j + \frac{1}{2}z''(0)(\delta j)^2 + O((\delta j)^3).$$

Note that $z''(0) = r^2 \varphi(r) \geq 0$. Respecting that $z(0) = \hat{z}$, we obtain $\delta z = \frac{z(\delta j) - \hat{z}}{\delta j} = \frac{z'(0)}{z''(0)}(\delta j)$. Thus $R^{\infty}_r(\varphi) = \frac{z'(0)}{z''(0)}$. Using $z(0) = \int_{\delta r}^{\infty} (y - r) \varphi(y) dy$, we obtain

$$R^{\infty}_r(\varphi) = \frac{\int_{\delta r}^{\infty} y \varphi(y) dy}{\int_{\delta r}^{\infty} \varphi(y) dy}$$

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    ylabel=$\delta z$,
    tick label style={font=\footnotesize},
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    grid style=dashed,
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Two ways to estimate sensitivities (100 events only!)

- **Empirical method**
- **Semi-analytic method**

![Graphs showing loss distribution and tail estimate](image)

- Estimated parameters:
  - $\hat{\alpha} = 2.6492$ (3) for empirical method
  - $R_{\text{fitted}} = 2.9369$
  - $R_{\text{analytic}} = 2.8498$
Is the Pareto $\alpha$ an universal upper bound?

Pareto Distribution

$$\phi_\alpha(y) = \frac{\alpha y^{\alpha-1}}{\gamma^{\alpha} \Gamma(\alpha)}$$

$\alpha > 1 \quad y \geq r$

Generalised Pareto Distribution

$$\phi(y) = \frac{1}{\sigma} \left(1 + \frac{y}{\sigma}\right)^{-(1+\alpha)}$$

$\alpha > 1$, $\sigma > 0$

Benktander-Weibull

$$\phi_{\alpha,b}(y) = \frac{d R_{\alpha,b}(y)}{dy}$$

Exponentially Distributed

$$\phi(y) = \alpha e^{-\alpha y}, \alpha > 0$$

$$R^\alpha_r(\varphi) = \alpha.$$
When is the Pareto $\alpha$ an upper bound for the vulnerability?
Sensitivity and Stabilisation clause


The relative change in unexpected payments $\delta z$ due to unexpected inflation $\delta j$ for a XL contract with retention $r$, cover $\kappa$ and a stabilisation clause whose clause index $k$ has rate $\delta k$ obeys

$$\delta z = \delta j + \left( R_r^\kappa(\phi) - 1 \right) \left( \delta j - \delta k \right)$$

The corresponding sensitivity yields

$$\mathcal{R}_r^\kappa(\delta k) = 1 + \left( R_r^\kappa(\phi) - 1 \right) \left( 1 - \frac{\delta k}{\delta j} \right)$$

The percentage change of sensitivity due to a stabilisation clause of clause index $k$ is

$$\delta \mathcal{R}_r^\kappa(\delta k) := \frac{\mathcal{R}_r^\kappa(\delta k) - R_r^\kappa(\phi)}{R_r^\kappa(\delta k)} = \frac{\delta k}{\delta j} \left( \frac{R_r^\kappa(\phi) - 1}{R_r^\kappa(\phi)} \right)$$
Thank you

“A major risk emerges if economic figures overrule ethical behaviour.”
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