General Public Release

Swiss Re

How sensitive is our exposure to inflation?

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On the role of inflation for (re) insurers

- BUSINESS Inflation erodes the underwriting discipline and profitability
  - Pricing
    - ‘*Calculate the premium required to cover all expect cost components*!’;

      Inflation increases the required premium, particularly for long-tailed business.
    - ‘The insurer receives a fixed premium *today* but has to compensate for the value of a claim in some *future*!’ A corresponding inflation caused gap would increase the price.
  - Demand for insurance
    - ‘*Buying insurance is forward-looking behavior*!’ In high-inflation regimes due to higher uncertainty, less mid-term and long-term investments are made.

      In the 1970s, higher inflation velocity coincides with subsequent drops in underwriting and profitability.
- RISK Unexpected inflation, deviation from the expected, is a risk for reserving.

Sensitivity: towards its definition in a One-Year-view

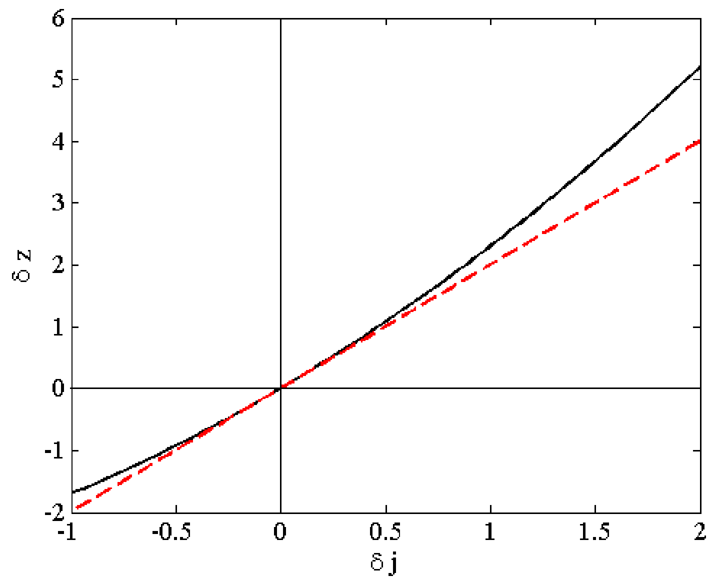
Given that at the end of the year inflation deviates from what is expected by n base point, how much does actual payment then deviate from the expected?

$$\delta j := \frac{j - \hat{j}}{\hat{j}}$$

expected inflation	$\hat{j}$	j	*realized inflation*
expected payment	$\hat{z}$	z	*realized payment*

$$\delta z := \frac{z - \hat{z}}{\hat{z}}$$

Sensitivity: its definition as an elasticity



DEFINITION [**Sensitivity**] The *sensitivity* of an exposure is defined as the elasticity of the payment function with respect to economic inflation, i.e.

$$\boxed{\mathscr{R} := \frac{\delta z}{\delta j}\bigg|_{\delta j = 0}}$$

Note that according to this definition, $z = \hat{z}(1 + \mathscr{R}\delta j)$ so that $\mathscr{R}\delta j$ might be regarded as the **rate of claims inflation**.

How to estimate the sensitivity of a contract from real data

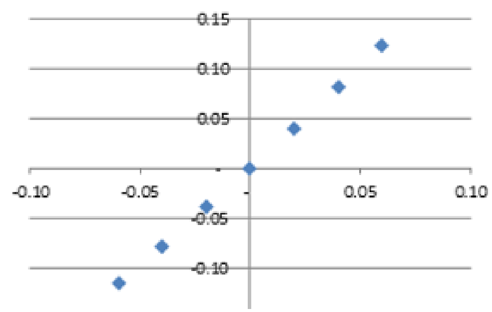
CONTRACT			STRUCTURED NOMINAL LOSS
LOSS MODEL			
severity	Pareto		
parameter	α	2	
ATTACHMENT POINT		2'000'000 USD	500'000 USD
COVER		oo	
further contract conditions including annual conditions		none	

Varying the inflation level is numerically equivalent to varying the deductible.

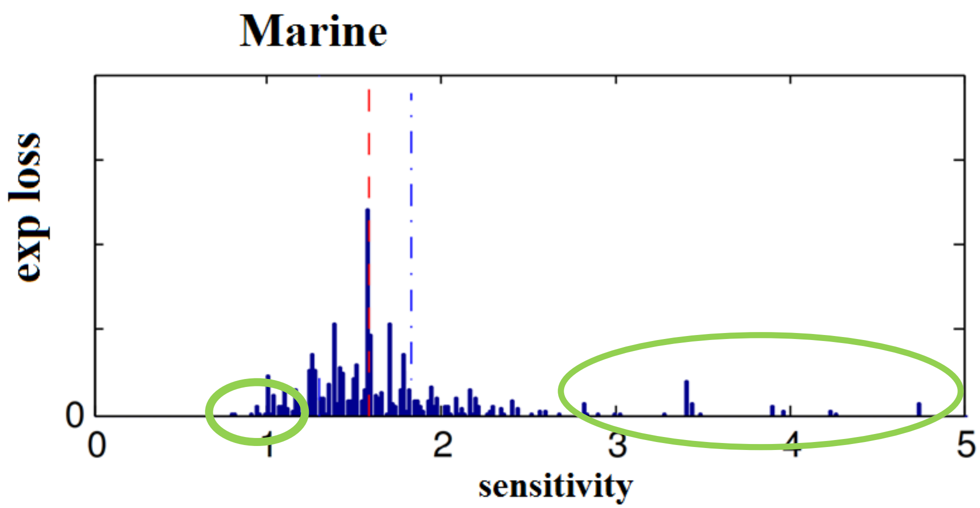
$$Z = \max(jX - d, 0) = j\max\left(X - \frac{d}{j}, 0\right)$$

$$\mathscr{R} := \left.\frac{\delta z}{\delta j}\right|_{\delta j = 0}$$

δj	j	d		z	δz	$\frac{\delta z}{\delta j}$	
delta j	j	mod att pt	mod struc loss	infl payment		ratio	sensitivity
-0.06	0.94	2'127'660	470'000	441'800	-0.12	1.94	
-0.04	0.96	2'083'333	480'000	460'800	-0.08	1.96	
-0.02	0.98	2'040'816	490'000	480'200	-0.04	1.98	
-	1.00	2'000'000	500'000	500'000	-		2.00
0.02	1.02	1'960'784	510'000	520'200	0.04	2.02	
0.04	1.04	1'923'077	520'000	540'800	0.08	2.04	
0.06	1.06	1'886'792	530'000	561'800	0.12	2.06	



Distribution of sensitivities in a Line of Business



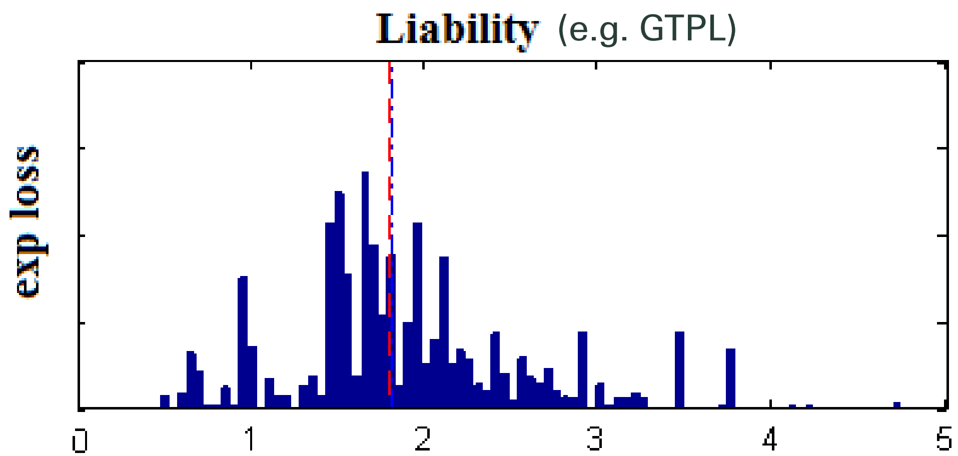
mean sensitivity: 1.80

median sensitivity : 1.58
MAD sensitivity: 0.20

The sensitivity of the portfolio is the convex sum of the individual sensitivties of the contracts involved

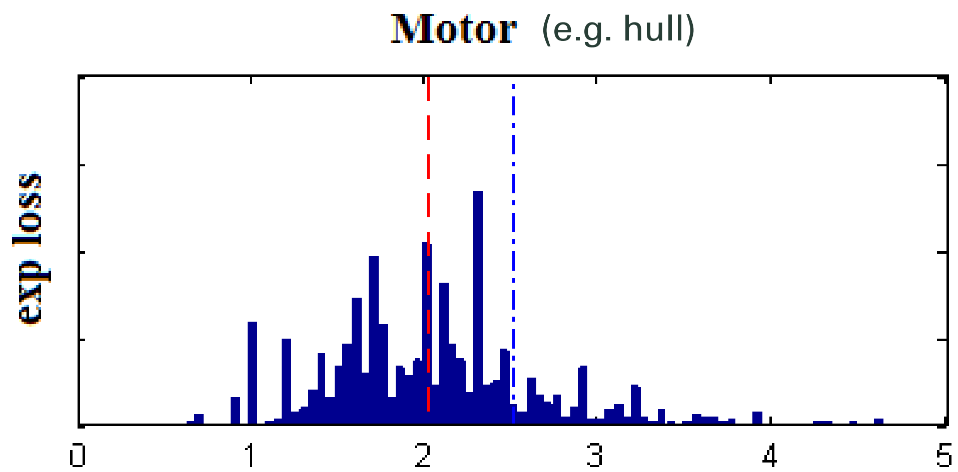
$$\mathscr{R} = \sum_i \zeta_i \mathscr{R}^{(i)} \qquad \zeta_i = \frac{z_i(0)}{\sum_j z_j(0)}$$

‘Experienced grown’ rule of thumb values and statistical estimate



RoTh:
range [1,2]
mean 1.8

median 1.8 (± 0.3)



RoTH:
range [1.5, 3]
mean 2.5

median 2.0 (± 0.4)

Back to the theoretical roots: What do these numbers mean

Let losses be distributed according to some distribution φ, whose first moment exists. Then the sensitivity of a Excess-of-Loss contract with retention $r \geq 0$ and infinite cover is

$$R_r^\infty(\varphi) \quad = \quad 1 + \frac{r}{\mathbb{E}[(Y-r)\,|\,Y \geq r]}$$

Let claim sizes C be distributed according to some φ, whose first moment exists. According to eqn (1) we have that $j = \hat{j}(1+\delta j)$, where $|\delta j| < 1$ is sufficiently small. Further let $X = jC$ and $Y = \hat{j}C$, so that $X = (1+\delta j)Y$. Expected payment is

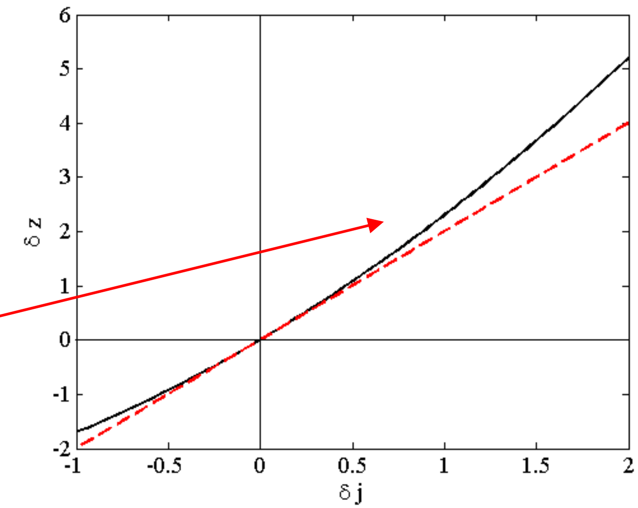
$$\mathbb{E}\,(X-r)^+ = z(\delta j) =: (1+\delta j)\mathbb{E}\left(Y - \tfrac{r}{1+\delta j}\right)^+$$

Expanding the expected payment in $\delta j = 0$ yields

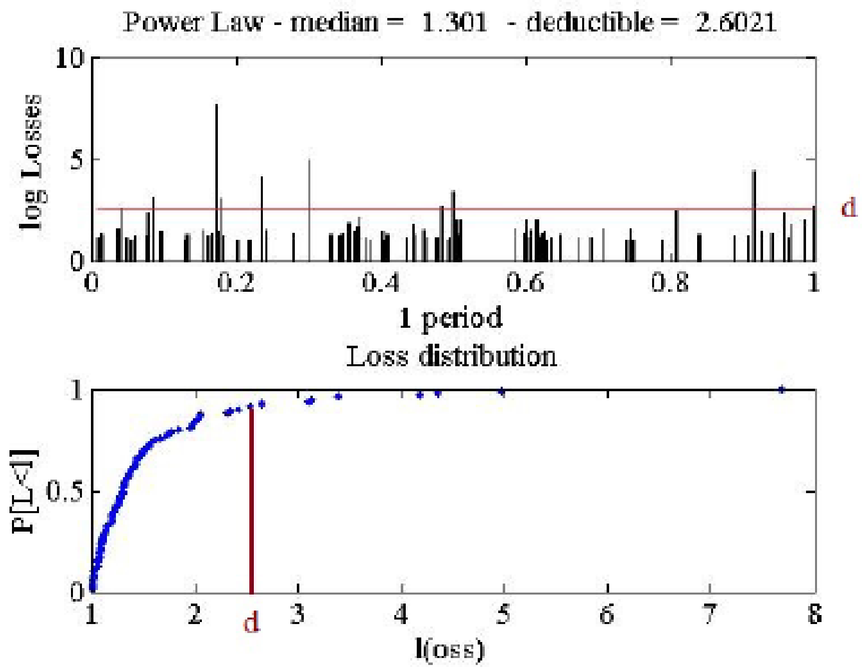
$$z(\delta j) = z(0) + z'(0)\delta j + \tfrac{1}{2}z''(0)(\delta j)^2 + \mathcal{O}\left((\delta j)^3\right).$$

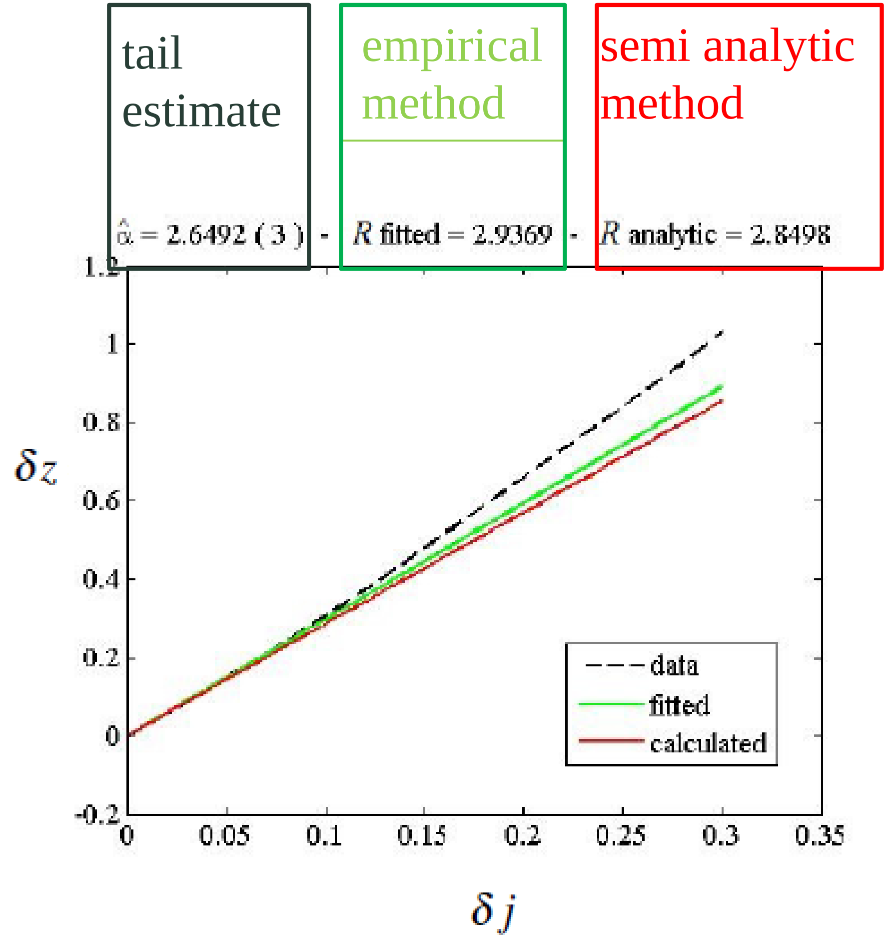
Note that $z''(0) = r^2\varphi(r) \geq 0$. Respecting that $z(0) = \hat{z}$, we obtain $\delta z = \frac{z(\delta j)-\hat{z}}{\hat{z}} = \frac{z'(0)}{z(0)}\delta j$. Thus $R_r^\infty(\varphi) = \frac{z'(0)}{z(0)}$. Using $z(0) = \int_r^\infty (y-r)\varphi(y)dy$, we obtain

$$R_r^\infty(\varphi) \quad = \quad \frac{\int_r^\infty y\varphi(y)dy}{\int_r^\infty (y-r)\varphi(y)dy}$$



Two ways to estimate sensitivities (100 events only!)





Is the Pareto α an universal upper bound?

PARETO DISTRIBUTION

$\varphi_\alpha(y) = \alpha \frac{r^\alpha}{y^{1+\alpha}}$

$\alpha > 1 \quad y \geq r$

$R_r^\infty(\varphi) = \alpha.$

GENERALISED PARETO DISTRIBUTION

$\varphi(y) = \frac{1}{\sigma}\left(1 + \frac{1}{\alpha}\left(\frac{y}{\sigma}\right)\right)^{-(1+\alpha)}$

$\alpha > 1, \ \sigma > 0$

$R_r^\infty(\varphi) = \frac{\alpha\sigma + \alpha r}{\alpha\sigma + r}$

BENKTANDER-WEIBULL

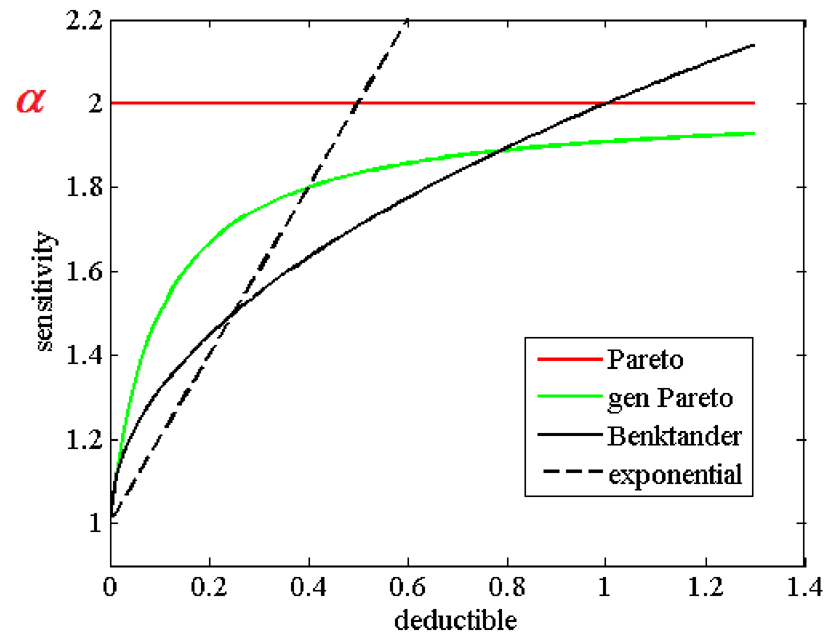
$\varphi_{\alpha,b}(y) \ = \ \frac{d B_{\alpha,b}(y)}{d y}$

$R_r^\infty(\varphi) = 1 + (\alpha - 1)\, r^b$

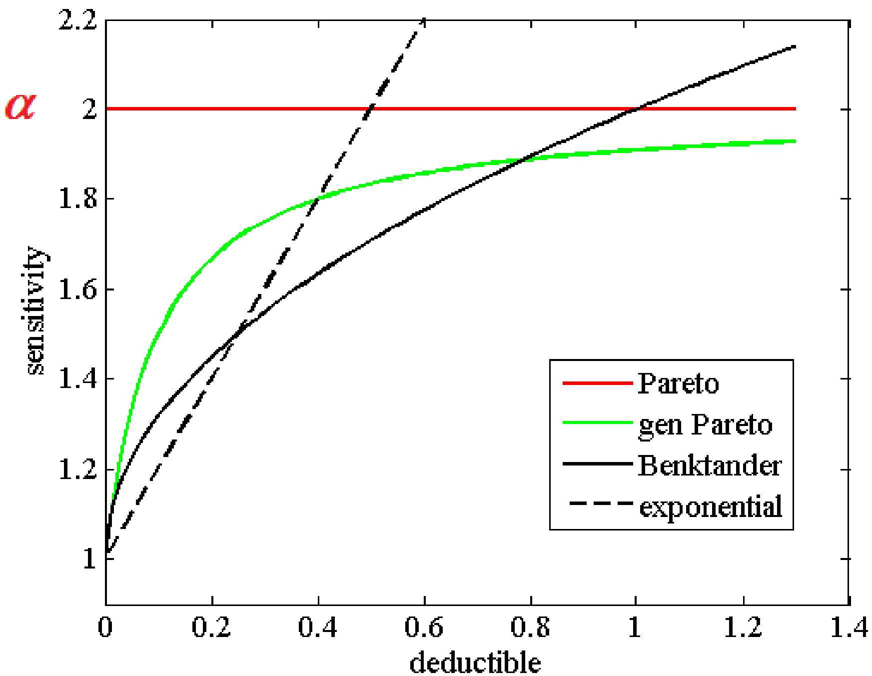
EXPONENTIALLY DISTRIBUTED

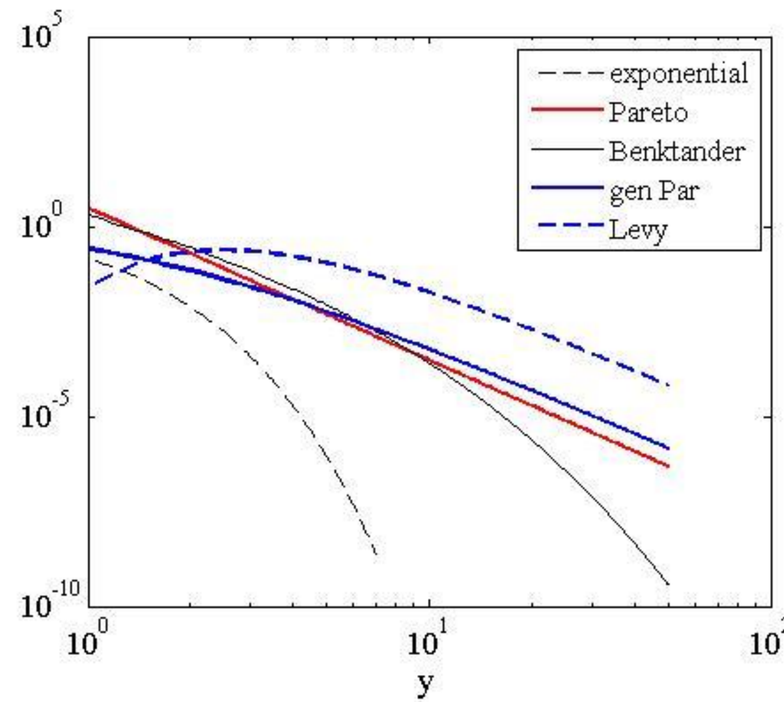
$\varphi(y) = \alpha\, e^{-\alpha y}, \ \alpha > 0$

$R_r^\infty(\varphi) = 1 + \alpha\, r$



When is the Pareto α an upper bound for the vulnerability?





Sensitivity and Stabilisation clause

(based on B.J.J. Alting von Geusau: Indexed Annuities and Stability Clause, General Insurance Convention 72(2), 163-174, 1985)

The relative change in unexpected payments δz due to unexpected inflation δj for a XL contract with retention r, cover κ and a stabilisation clause whose clause index k has rate δk obeys

$$\delta z = \delta j + \left(R_r^\kappa(\phi) - 1 \right) \left(\delta j - \delta k \right)$$

The corresponding sensitivity yields

$$\boxed{\mathscr{R}_r^\kappa\left(\delta k\right) = 1 + \left(R_r^\kappa(\phi) - 1 \right)\left(1 - \frac{\delta k}{\delta j} \right)}$$

The percentage change of sensitivity due to a stabilisation clause of clause index k is

$$\delta \mathscr{R}_r^\kappa\left(\delta k\right) := \frac{\mathscr{R}_r^\kappa\left(\delta k\right) - R_r^\kappa(\phi)}{R_r^\kappa(\delta k)} = -\frac{\delta k}{\delta j}\left(\frac{R_r^\kappa(\phi) - 1}{R_r^\kappa(\phi)} \right)$$

Thank you

"A major risk emerges if economic figures overrule ethical behaviour."

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