Nested Stochastics in Life Insurance

What is “nested stochastics” and why do you need “nested stochastics”
The crucial steps to make nested stochastic simulations feasible
Accurate fast models increase the transparency of the risk exposure
Why “nested stochastics” is the most efficient method for solvency and ALM
Embedding all solvency calculation techniques in a unifying framework

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Dr. Urs Burri
Leiter Aktuariat & Steuerung
Basler Leben AG
Basler Versicherung AG

Making you safer.
Core of all modern valuation frameworks: MCEV engine

- Economic scenarios
- Financial data / Assets
- Policy data / Liabilities
- Biometric parameters
- …..

Properties: Dynamic model
- Various interactions/Loops
- Stochastic calculation
- Complex

- Accurate calculation of all embedded options and guarantees
- Slow

Market consistent valuation e.g. MCEV
IFRS/Local GAAP/Business plan
Fair value balance sheets for:
SST and Solvency II
Core of all modern valuation frameworks: The Baloise case

Properties:
- Realistic management decision rules with good backtesting results
- ALS with monthly exact "external liabilities"¹
- e.g. individual life: 48 dynamic segments

Very slow: MCEV calculation (5'000 simulations)
- individual life on 70 CPUs: 8 h
- group life on 70 CPUs: 20 minutes

¹ SUNGARD iWorks Prophet
Projection of market consistent balance sheets

Motivation: Assessment of risk exposure requires distribution of risk bearing capital (RBC)

Goal: Accurate calculation of the distribution of the RBC at \( t = 1 \)

Embedded options and guarantees \( \rightarrow \) Stochastic calculation

\[ t = 0 \quad t = 1 \]

- Risk free interest
- FX EUR-CHF
- Credit spread

10-20 different market risk factors

- Real world scenarios \( S \)

Reasonably accurate calculation of expected shortfall (1%) requires about 100'000 ("outer") real world scenarios.

Runtime of calculation becomes prohibitive

\[
\begin{align*}
\text{ind. life: } & 100'000 \times 500 \times 8h/5000 = 80'000 \text{ h} \approx 475 \text{ weeks} \\
\text{group life: } & 100'000 \times 500 \times 1/3 h/5000 = 3'333 \text{ h} \approx 20 \text{ weeks}
\end{align*}
\]
Common approach: Use of replicating portfolio (RP)

**Strategy:**
- Ignore complexity of model (e.g. management rules)
- Find replicating portfolio for liability CFs: \( \text{RP}(L) = \sum \text{candidate assets} \)
- Candidate assets: ZCB, puts, calls, swaptions, …

**Requirements:**
\[
\frac{\partial}{\partial \text{risk factor}} \text{PresentValue}(L) \approx \frac{\partial}{\partial \text{risk factor}} \text{PresentValue}(\text{RP}(L))
\]

- Candidate assets analytically tractable
- Solvency capital calculation straightforward

**Challenges:**
- Hard to find good RP\(^2\)
- Criteria for goodness-of-fit, metric\(^2\)
- Overfitting: Subspace of null vectors is huge\(^2\)
- Quality of roll forward of \( \text{RP}(L), \text{quality of RP}(L) \) in tail
- Impossible to replicate cash flows of path dependent options (e.g. rolling means, legal quote, bonus philosophy) with path independent candidate assets

**Our conclusion:** Establishment of reliability of RP would be a formidable task \( \rightarrow \) Look for an alternative

\(^2\) Presentation J. Crugnola/A. Meister: Stochastic Uncertainties in Market Consistent Valuation
Crucial steps to make nested stochastic simulation feasible

- Try to solve a **harder problem**!
- Find a fast tool that **replicates** all cash flows in every projection year.
- Solve the **right** problem! Do not ignore the **dynamics of the “fund”**.
- In a first step it might help to integrate **replicating portfolio techniques** into the dynamic model

- Then think about the **run-time-inefficiencies** in your models, look at a TEV-calculation for example…
- Analyse your **MCEV model** in detail
- In a further step replace in your “vision“ the word “proxy” by “**perfect analytical / algebraic replication**”

This analysis requires a **deep understanding** of your MCEV model, on the actuarial as well as on the **soft- and hardware** level.

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3 Case study with SUNGARD
Our approach: Construction of a fast model

Market consistent Balance Sheet

MCEV model: Prophet ALS

Generic framework: accommodates wide range of applications

Fast model: In Prophet

Dedicated framework: trimmed to speed up specific applications

Assets

driven by economic scenarios

mapped to ALS classes

grouped

Liabilities

full-fledged model

Fund

Mgt Decisions

PH-Behavior Accounting

RBC

speed up

MV(A)

BEL

further grouping yearly cash flows

Aim: Replication of cash flows in every scenario in every year.

MCEV model

Fast model

Full complexity of path-dependent embedded options is retained

"Stochastic commutation functions"

SH Cash Flows

RBC

Generic framework: accommodates wide range of applications

Dedicated framework: trimmed to speed up specific applications

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MCEV model

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"Stochastic commutation functions"
Quality test of fast model: Group life

Comparing sample valuations at \( t = 1 \) using the "MCEV model" and the "fast model". Sampling over the full range of the distribution of RBC at \( t = 1 \):

\[ \text{Run-Times per sample valuation (500 Simulations, 20 CPUs):} \]
\[ \bullet \text{"MCEV model": ca. 7 min} \]
\[ \bullet \text{"Fast model": ca. 0.2 s} \]
\[ \rightarrow \text{Speed-up by a factor of more than 1'000} \]

\[ \rightarrow \text{The quality is uniform over the whole range of scenarios from worst to best.} \]
Advantages of nested stochastics techniques: Modeling Aspects I

- Conceptually very clean method
- Sound economic interpretation of the results
- 50 Mio. simulations in 12 h instead of 475 weeks (for individual life)
- Calculation of distribution of \( RBC_{t=1} \) in 12 h → \( \text{VaR}_\alpha \), \( \text{ES}_\alpha \) straightforward
- Validation "fast" versus "MCEV model" in 2-3 days
- Calibration, i.e. transformation of "MCEV model" → "fast model" with fast robust algorithm, no "art" or "trial and error"
Advantages of nested stochastics techniques: Modeling Aspects II

- Dynamic model calculation without grouping of model points seems to be accessible. Main problem: memory (de)allocation

- "Fast model" completely equivalent to "MCEV model" (e.g. update of management decision report with over 160 sensitivities)

- The group life model allows for an analytical and economically interpretable basis of "candidate liabilities" (summarized in 3 pages of concise formulae)

- Whether 50 Mio. simulations are sufficient or not is an inherent problem of all stochastic techniques. An acceleration of O(1) is always possible by brute force hardware improvements.
Advantages of nested stochastics techniques: ALM aspects

- The "fast models" are complete ALM-tools
  If you change tactic or strategic asset allocation, there is *no need* for a recalibration
- ALM and SST calculations are "consistent"
- Good platform for developing **optimal hedging strategies**
- Efficient "risk dimension reduction methods" can be implemented

→ Focus on the tail
Advantages of nested stochastics techniques: Solvency aspects

- Easy to verify other proxy methods\(^4\)
- Confidence intervals for expected shortfall easily accessible:
  
  Apply standard statistical methods to the tail of the pertinent distribution, which is conveniently described by generalized Pareto-distribution\(^5\)

- Other model enhancements:

  Replace 1 year by \(\tau\) 1 year

  \(\tau = 1, 2, \ldots, 12\) months: monthly SST
  
  \(\tau = 1, 2, \ldots, 5\) years:
  
  ORSA-like solvency reporting

\(^4\) Ambrus et al., *Interest Rate Risk: Dimension Reduction in the Swiss Solvency Test*

Embedding all Solvency calculation methods in one unifying framework

- Initial data (Policy and Asset Data): \( m = (m_1, m_2, \ldots, m_p) \in \mathbb{R}^p \)
- Scenario data: \( \sigma = (\sigma_1, \sigma_2, \ldots, \sigma_s) \in \mathbb{R}^s \)
- Contribution of one simulation path \( \sigma \) to the RBC after one year: \( RBC_1(\sigma, m) \)
- Tensor Product Decomposition\(^6\): \( F(\mathbb{R}^s \times \mathbb{R}^p) \ni RBC_1(\sigma, m) \approx \sum_j c_j(\sigma) \cdot B_j(m) \in F(\mathbb{R}^s) \otimes F(\mathbb{R}^p) \)
  (exact equality may require "infinite sums")

In principle, all "proxy techniques" used for Solvency calculations can be written in this way, e.g.:

- SST Standard model ("Delta-Gamma") and other "formula fitting" methods:
  \[
  RBC_1 = RBC_0 + \sum_l \Delta r_l \cdot B_l^{(1)} + \frac{1}{2} \sum_{k,l} \Delta r_k \Delta r_l \cdot B_{kl}^{(2)} + \ldots
  \]

- RP: For fixed \( m \), the \( c_j(\sigma) = A_j(\sigma) \) are the candidate assets, the \( B_j = B_j(m) \) are the coefficients determined by regression:
  \[
  RBC_1(\sigma, m) \approx \sum_j A_j(\sigma) \cdot B_j
  \]

- Our nested stochastics approach: \( c_j(\sigma) \) and \( B_j(m) \) are both determined algebraically using algorithms.
  No fitting of coefficients is involved as in other approaches.

\(^6\) Stone-Weierstrass theorem
Conclusions

➢ Full nested stochastics is challenging but feasible

➢ Prerequisites: Understand your model, your software and your hardware

➢ Once the right problems are identified and solved, the model structure is preserved in full integrity: Powerful and transparent method for ALM/solvency considerations

➢ Access to the distribution of RBC opens the door to a plethora of new applications

➢ Nested stochastics models allow one to address the relevant task: Providing concise information for the management to drive decisions
References

- Case Study SUNGARD,
  http://www.sungard.com/~media/financialsystems/casestudies/iworks_casestudy_baloise.ashx


  http://dx.doi.org/10.2139/ssrn.1935378

Evolution of valuation frameworks & software @Baloise.Switzerland


Embedded value
- PVFP
- TEV publ.
- MCEV proj.
- MCEV publ.

Accounting
- Local GAAP LAT
- IAS / US GAAP
- IFRS 4 Phase I
- IAS 19
- ..IFRS 4 Phase II

Swiss solvency test
- SST Fieldtest
- SST Standard Model
- SST internal model

Solvency II
- QIS 4 Standardmodell
- QIS 5 internal model

Software
- Prophet international
- Test Asset/Global
- Test Life DFA
- ALS ext. liab.
- RP Tool

Hardware
- PC
- Grid
- Server 1 20 CPUs
- Server 2 24 CPUs
- Server 3 32 CPUs

Mathematics
- deterministic
- stochastic calc.
- market cons. val.
- RP
- nested stochastic
Confidence intervals for expected shortfall:

- **Theorem of extreme value theory:**

  For a large class of distributions $F$ of the variable $X$, the distribution of the excess losses $F_u$ over the threshold $u$,
  $$F_u(y) = P[X - u \leq y | X > u]$$

  converges to a **generalized Pareto distribution** $G_{\xi,\beta}$ with parameters $\xi$ and $\beta(u)$.

- **The pertinent tail of the distribution of $-\Delta RBC$ can accurately be described by a generalized Pareto distribution**, given the estimate $F(u) \approx (N - N_u) / N$ with $N_u$ the number of data points with values above threshold $u$ in a sample of size $N$. VaR$_\alpha$ and ES$_\alpha$ have the simple form

  $$\text{VaR}_\alpha = u + \frac{\beta}{\xi} \left[ \left( \frac{N}{N_u} (1 - \alpha) \right)^{-\xi} - 1 \right], \quad \text{ES}_\alpha = \frac{\text{VaR}_\alpha + \beta - \xi u}{1 - \xi}.$$ 

- **Standard statistical methods** allow for a **straightforward determination** of confidence intervals of VaR$_\alpha$ and ES$_\alpha$ for $\alpha$ close to 1 (in our case $\alpha \geq 0.92$ roughly).

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Contact:

Basler Versicherungen
Dr. Urs Burri
Aeschengraben 21
CH-4002 Basel
urs.burri@baloise.ch