B. Wissenschaftliche Mitteilungen

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SOLVENCY – a historical review and some pragmatic solutions

1 Introduction

The *solvency margin* is a buffer in a company's assets that covers a theoretical capital required by the regulator and *solvency* is the ability of an insurance company to pay future claims as they fall due. This involve that the insurer must have sufficient assets to meet the liabilities but also to satisfy statutory financial requirements. For the supervisor it is important that the policyholders are protected. But it is also important for the supervisor to ensure the stability on the financial market.

The Available Solvency Margin, ASM, is the difference between assets (A) and liabilities (L). This definition, in terms of solvency margin, was first given in Pentikäinen (1952). We must distinguish between this actual, available solvency margin and a theoretical, by the regulator required, solvency margin. The former is the real value as defined above and the latter a theoretical amount required by the regulator in its protection of the policyholders or set by the insurer itself for its internal control.

The theoretical capital requirement, TCR, can in some jurisdictions be the minimum amount required by the regulator so that the insurers can continue its business in some form. In other jurisdictions the TCR is just a target or an early warning signal. This means that if the insurer has an ASM above the TCR it can continue as a going concern. Otherwise it has to be a dialogue between the insurer and the supervisor on what steps to take to be sure that the ASM is above the TCR. Some systems[1] have an intervention ladder between the upper target level and the absolute minimum level.

In the sequel we will assume a jurisdiction with two regulatory capital requirements[2]; the target will be called the Solvency Capital Requirement, SCR, and the absolute minimum requirement will be called the Minimum Capital Requirement,

[1] e.g. the National Association of Insurance Commissioners' (NAIC) risk-based system in the US

[2] If we only have one level of requirement, then $\mathrm{SCR} = \mathrm{MCR}$.

MCR. Ideally we have

$$\text{MCR} < \text{SCR} \leq \text{ASM}\,. \tag{1}$$

The definition of solvency given in Benjamin (1977) gives rise to two concepts of solvency, the two extremes of a range of possibilities:

- the liabilities are those paid on an immediate liquidation of the company (a break-up or a run-off of the company) or if its liabilities could be transferred to a willing partner, or
- the company is regarded as solvent if it pays all its debts as they mature (the going concern approach).

The first position may be obtained when $\text{ASM} \leq \text{MCR}$, i.e. when the insurer breaks the minimum floor the supervisor will intervene and decide if the company must break-up (all business are closed) or if the business should be put in run-off (no new business is allowed, but all old contracts are to be fulfilled). The second position may be obtained when $\text{ASM} \geq \text{SCR}$. In Campagne (1961) the term dynamic solvency was used for the going-concern approach and static solvency for the break-up situation, see also Kastelijn & Remmerswaal (1986).
Note that the liability concept refer to the obligations set out in the insurance contracts. The technical provision is the value of the insurance obligations set aside in the balance sheet. Traditionally the technical provision includes implicit margins of prudence.
By use of risk theory techniques the strength of solvency can be evaluated, (cf. also Pentikäinen (2004)). The ruin probability is the probability that the insurer, having an initial available solvency margin $\text{ASM} \geq \text{SCR}$, will become insolvent during a chosen time horizon $(0, T]$. With insolvent in its legal meaning we mean the break of the MCR within the time interval $(0, T]$. T is usually, at least, one year, and is usually chosen according to the accounting period.
In the literature, see below, there are a number of different formulas for the assessment of the capital requirement, especially for the MCR. In the beginning of the 1990s a tendency was to seek rules that take into account all risks that the insurance companies are facing. Systems of this kind are usually called risk based and hence a risk based capital requirement. A thorough discussion of the insurers' risks is given in IAA (2004), see also Sandström (2005).
In constructing a solvency capital requirement we need, at least, discuss these fundamental issues:

- *valuation* of assets and liabilities
- risk margins for *uncertainty* in liabilities and assets

- risk *measures*.
- the *modelling* (risk categories, risk mitigation, diversification etc)

Before doing this we will summarize the historical treatment of solvency.

2 Historical treatment of solvency

The pioneering works done by Cornelis Campagne in the Netherlands at the end of the 1940s and by Teivo Pentikäinen in Finland in the beginning of the 1950s are important, as they introduced the solvency research for insurance undertakings, see e.g. Pentikäinen (1952), Campagne (1961), and Campagne, van der Loo &. Yntema (1948).

Before the term solvency was introduced, a concept like statutory reserves was often used, "which have been formed in the course of years and which serve as an extra guarantee for fulfilling the obligations undertaken" [Campagne et al. (1948, p. 338)]. Initially, Campagne called this type of reserve for life insurance for a stabilization reserve. In Finland a special equalization reserve was introduced in 1953 to take account of the stochastic fluctuations in the annual claims amount in non-life insurance. During the 1950s Campagne enlarged the solvency assessment to non-life insurance.

As Campagne's work became leading for the approach of assessing an extra minimum reserve for both life and non-life companies he was asked to present a report on solvency ("Minimum Standards of Solvency for Insurance Firms") in 1957 to the OEEC[3] Insurance Committee. As a chairman of a working group within the Insurance Committee his work was developed and a final report was presented in 1961, Campagne (1961).

2.1 Campagne's work: Life insurance

The approach adopted was the same as in the 1940s. As the risk on investments is the most important factor for life insurance companies and as the technical provisions are the most important invested amount, Campagne considered a minimum solvency margin as given by a percentage of the technical provisions, see Campagne (1961), Kastelijn & Remmerswaal (1986), Campagne et al (1948), and Willemse & Wolthuis (2005). Campagne asked "how great has the extra

[3]Organization for European Economic Cooperation, now OECD the Organization for Economic Cooperation and Development.

reserve to be, so that with a probability smaller than 1/100 respectively 1/1000 this can be expressed to be insufficient for the financing of investment losses and deviations of foundations; in which case furthermore distinctions have to be made between cases in which the stabilization reserve has to be sufficient for one year or more years." [Campagne et al (1948), pp. 342–343]. A Pearson type IV distribution seemed to fit data best. Campagne concluded that an extra reserve of 6% of the technical provision would be adequate with a probability of 99%. With a probability of 95% the percentage of the extra reserve became 4% and this was the extra reserve proposed by Campagne. It was implemented in the first life directive within the European Union[4] in 1979.

2.2 Campagne's work: Non-life insurance

The model was simple but elegant. Let the net retained premium be 100%. From this we deduct a constant fraction equal to the average expense ratio (fixed to 42%). The remaining part is what remains for claims payment. Campagne assumed that the net loss ratio followed a beta distribution. With data from different European countries he estimated the Value-at-Risk of the loss ratio at 0.9997% as 83%. Thus the combined ratio will be $42\% + 83\% = 125\%$. In other words the company will need an extra 25% of the premium during 1 year to meet the requirements. After further works during the 1960s and political negotiations this framework became the base for the first non-life directive in Europe in 1973.

2.3 Other works

Research on solvency assessment was initiated as many countries in Europe had got the non-life and life directives during the 1970s implicating minimum solvency margins. Work was done in e.g. United Kingdom (Daykin (1984), Daykin et al (1984), Daykin et al (1987), Daykin & Hey (1990)), the Netherlands (Kastelijn & Remmerswaal (1986), Wit & Kastelijn (1980)), but also in Finland (Pentikäinen (1982), Rantala (1982), Pentikäinen et al (1989)), and Norway (Norberg & Sundt (1985), Norberg (1986) and Norberg (1993)). The best reference and summary of different solvency assessment methods used to the middle of the 80s is given in Kastelijn & Remmerswaal (1986).

The research and works done were all stepwise towards a risk based capital (RBC) approach. The NAIC introduced a RBC-system for life and health insurers in 1992

[4] At that time the EEC, the European Economic Community.

and for non-life insurers in 1993. At the same time the Canadian Office of the Superintendent of Financial Institutions (OSFI) introduced a risk based system in 1992 for life insurers. The system was in one way static and later amended and made more dynamic. Risk based systems were also discussed and introduced in Australia, Singapore and Japan and within the European Union. At the same time, waiting for the European system, different solvency assessment systems were introduced in United Kingdom, Switzerland and the Netherlands. Traffic light systems based on stress tests were introduced in Denmark and Sweden (from the beginning only as a supervisory tool); see e.g. Sandström (2005).

Historically, there have been problems in comparing the available and required solvency margins between companies (and especially between companies in different countries). Assets have either been defined as historical book values or as market values. But the main problem has been the technical provisions as these have included implicit margins to protect policyholders. These implicit margins have been set by the actuaries and have been reflecting the prudence of the company. Even in the European Union, with its life and non-life insurance directives, the incomparability of the technical provisions have been recognised and discussed.

The works done by Campagne were the base for the solvency directives within the European Union, see above. The first solvency directives from the 1970s have been amended in the second and third directives from the 1980s and 1990s. Based on the discussions in the Müller report (1997) the EU Parliament in 2002 adopted revised directives, Solvency I, and at the same time worked on a future risk-based system[5]. This new solvency system will be implemented within Europe around 2010. The basic ideas of Solvency II are given in EU Commission (2006).

3 Risk-based systems

The International Actuarial Association, IAA, and the International Association of Insurance Supervisors, IAIS, have issued standards and guidance regarding the assessment of insurer solvency[6]. In IAIS (2006b) the IAIS summarize the main concepts of the solvency assessment.

"*A total balance sheet approach should be used to recognise the interdependence between assets, liabilities, capital requirements and capital resources and to*

[5]For more information on the Solvency II project, see `http://ec.europa.eu/internal_market/insurance`.

[6]For the latest versions, see the two associations' websites. IAA: `www.actuaries.org`, IAIS: `www.iaisweb.org`.

ensure that risks are fully and appropriately recognised." (Structure Element 4, IAIS (2006b))

"A risk sensitive solvency regime should require insurers to assess and manage the risks to which they are exposed and appropriately assess and maintain their capital needs. By requiring this, supervisors can effectively achieve their aims of protecting policyholders and maintaining well-founded market confidence. These aims require adequate levels of capital and this in turn requires that risks are measured properly. Regulatory financial requirements therefore need to be firmly rooted in economic valuation and provide the basis and incentives for optimal alignment of risk management by the insurer and regulation. Regulatory financial requirements should be as complete as practicable, i.e. include all risk factors that can be appropriately translated into a financial requirement." (IAIS (2006b))

The total balance sheet approach was introduced by IAA, see IAA (2004), and should not only recognise the asset and liability sides of the balance sheet, but also the interdependence between them and the impact of the SCR, MCR and the eligibility of capital covering the requirements. The technical provisions are the reserves set aside to cover the liabilities the company face according to the insurance contracts. It will usually also include a risk margin, cf. below.

In the banking supervisory system, Basel II, a three-pillar approach was introduced. The European Union Solvency II system has also a similar approach. It consists of

Pillar I: The quantitative requirements

Pillar II: The qualitative requirements; the supervisory review process

Pillar III: Statutory and market reporting.

The first pillar includes the calculation of the SCR according to a standard model (e.g. factor based), or the introduction of partial or full internal models. It also includes rules on provisioning and eligible capital. The second pillar is focusing on the supervisors and their review process, e.g. a company's internal control and risk management, the approval of using partial or full internal models in Pillar I and its validation. The supervisor can also impose a company to increase its SCR, so called add-ons, if it believes that the capital is not adequate or that the management is insufficient. The third pillar includes the reporting to both the supervisor and the market. The latter case will promote the market discipline and greater transparency, including harmonization of accounting rules.

"*A risk sensitive solvency regime could use some or all of the following:*

- *regulatory financial requirements, ranging from sophisticated risk sensitive requirements to simple rations or even nominal minimum requirements including necessary safety measures*
- *quantitative limits to risk exposures*
- *qualitative requirements*
- *additional quantitative and qualitative capital requirements arising from supervisory assessment."* (IAIS (2006b))

The first two bullets correspond to the first pillar and the last two to the second pillar. Parts of pillar III issues are set out in the following sentence.

"Public disclosure of information enhances market discipline, imposing strong incentives on insurers to conduct their business in a safe, sound and efficient manner. Insurer solvency and solvency assessment thus benefit from appropriate public disclosure. A regime would be expected to differentiate between public disclosure and reporting to the supervisor." (IAIS (2006b))

3.1 Valuation of assets and liabilities

"Regulatory financial requirements therefore need to be firmly rooted in realistic economic valuation ...." (IAIS (2006b))

A total balance sheet approach is based on common valuation methodologies. It should make optimal use of information provided by the financial markets, EU Commission (2006), in getting market values where they exist or getting market consistent values[7] where market values don't exist, of both the assets and the liabilities. This is called the economic value[8]. In a traditional actuarial valuation of present value a deterministic interest rate function is used. In market consistent valuation the deterministic interest rate is changed for a stochastic function, a deflator, reflecting the market price. The valuation should be prospective and all cash flows related to assets and liabilities should be discounted and valued at current estimate[9]. The expected present value of the future cash flows should

[7]The value of assets and liabilities based on market values where available (mark-to-market), where not, on market-consistent valuation techniques (mark-to-model), see CEA-Groupe Consultatif (2006).

[8]Economic value: The value of assets or liability cash flows, derived in such a way as to be consistent with current market prices where they are available or using market consistent principles, methodologies and parameters, see CEA-Groupe Consultatif (2006).

[9]Current estimate: the discounted mean value of future cash flows. In Australia the term central estimate is used and in the European Solvency II project best estimate. Current estimate is defined by IAA and IAIS.

use a relevant risk free yield curve and should be based on current, credible information and realistic assumptions. A risk margin covering the uncertainty linked to future cash flows over their whole time horizon is added to the current estimate. A discussion on risk margins is given in Mourik (2005).

The risk adjusted current estimate of the liabilities is called "economical" technical provisions, or market value of liabilities, MVL. The corresponding risk adjusted assets are called market value of assets, MVA. The risk margins should be determined in a way that enables the insurance obligations to be transferred to a third party or to be put in run-off, cf. e.g. EU Commission (2006).

To hedge means to offsetting the risk inherent in any market position by taking an equal but opposite position in the market. Thus, any loss on the original investment will be hedged, or offset, by a corresponding profit from the hedging[10] instrument. Hedging has become an important and accepted risk management tool in risk mitigation, see below.

In the valuation procedure hedgeable assets and liabilities should be valuated by a mark-to-market approach, as any risk margins are implicit in observed market prices. Non-hedgeable assets and liabilities should be valued by a mark-to-model approach, i.e. a current estimate requires the calculation of an explicit risk margin. This is illustrated in Figure 1.

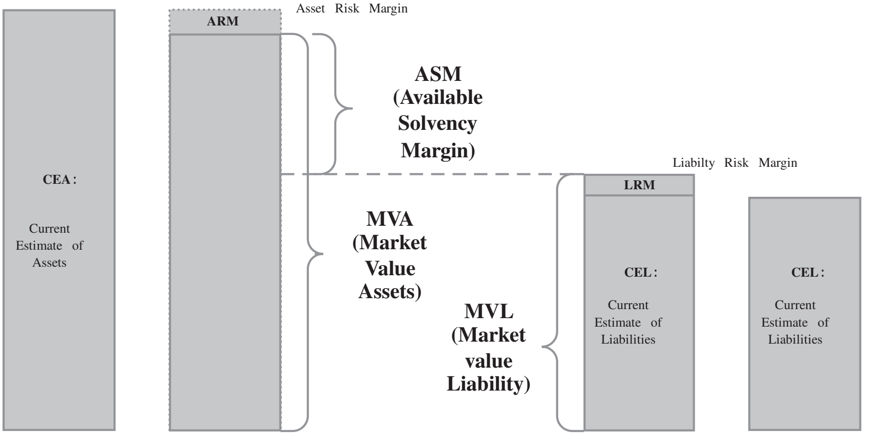

Figure 1. Current estimates of assets and liabilities are calculated, CEA and CEL respectively. An asset risk margin, ARM, is then deducted from the CEA to get the market values of assets, MVA. A liability risk margin, LRM is then added to the CEL to get the market value of liabilities, MVL. *Source*: Sandström (2006b)

[10]One way of thinking of hedging is to think in terms of insurance. If you decide to hedge, you are insuring yourself against a negative event. Hedging is a technique by which you will not make money, but by which you can reduce a potential loss.

In terms of a total balance sheet approach the available capital ("available solvency margin") should be written as

$$\mathrm{ASM} = \mathrm{MVA} - \mathrm{MVL}\,. \tag{2}$$

Asset risk margin, ARM:
The asset risk margin is a margin deducted from the current estimate of assets due to uncertainty in models and parameters. Valuations of derivative instruments may include uncertainty in the models used see e.g. Cont (2006). Valuation of property depends very much how frequently it is made. Yearly valuation may give accurate calculation of the property value, but if you do daily valuations you need some model behind the calculation; this gives rise to an uncertainty in the valuation due to models and parameters.
One way in calculating the ARM is to take a proportion λ, $0 < \lambda < 1$, of the spread, i.e. the difference between the ask price, C^{ask}, and the bid price[11], C^{bid}, i.e. $\mathrm{ARM} \approx \lambda \left| C^{\mathrm{ask}} - C^{\mathrm{bid}} \right|$. A coherent measure of model uncertainty is given in Cont (2006).

Liability risk margin, LRM:
As there is no liquid market for insurance risks, we need to use a mark-to-model approach to determine the MVL, i.e. to model a risk margin on top of the current estimate of liabilities, CEL, see Figure 1. It should take account of the uncertainty of models, parameters and be such that the insurance contracts could be sold to a "willing buyer" or put in run-off. In economic terms the risk margin is often called a market value margin.

In Australia, the risk margin for non-life insurance is calculated as the 75^{th} percentile of the distribution function where the unbiased mean equals current estimate of liabilities. Using an economic approach, a proxy of the LRM (or market value margin) can be given by a cost-of-capital, CoC, approach. "The cost of capital approach bases the risk margin on the theoretical cost to a third party to supply capital to the company in order to protect against risks to which it could be exposed", CEA (2006), see also CEA-CRO Forum (2006). Market value of mortality risk is discussed in van Broekhoven (2002).
The CoC-approach was first introduced in the solvency context in the Swiss Solvency Test, see SST (2004) and Sandström (2005), where the risk margin is defined as the hypothetical cost of regulatory capital necessary to run-off all

[11]The ask price, or offer price, is the price a seller of a commodity is willing to accept for it and the bid price is the price offered by a buyer (bidder) when he buys the commodity. The difference between these prices is referred to the "bid-ask spread" or just spread.

liabilities, following financial distress of the company. Let SCR_t denote the capital requirement for year t, $t = 1, \ldots, T$, for the liabilities in run-off. Then the risk margin is calculated as[12] $\mathrm{LRM} \approx \mathrm{CoC}\% \sum_{t=1}^{T} \mathrm{SCR}_t$, where SCR usually is discounted with its time value.
An alternative in using a deflator function in market consistent valuation is to use a replicating portfolio in valuing the liabilities, i.e. a portfolio of assets that replicates the cash flow of liabilities most closely. One such technique is to use a Valuation Portfolio (VaPo), see Bühlmann (2002), (2003), (2004), and Wüthrich et al (2006).
The IAIS, (IAIS (2006b)), has suggested that risk factors and their distinct components should be represented in the technical provisions, i.e. in the LRM within the MVL see Figure 1, and in the capital requirements, SCR, as follows:

- Risk to be reflected both in in the LRM and the SCR:
  - Uncertainty and residual market volatility in underwriting risk
  - Unhedgeable mismatch risk
- Risk that is reflected only in SCR and not in the LRM:
  - Volatility other than residual market volatility in underwriting risk
  - Hedgeable mismatch risk.

On way of distinguishing between risks taken care of in the risk margin, LRM with a time horizon to the ultimate, and the solvency capital requirement, SCR with a short time horizon, say $T = 1$ year is to use the mean square function.

3.2 Different elements of the solvency assessment system

We illustrate the different elements of the supervisory system and requirements in the following way. Let $F = \sigma\{M, \Theta\}$ be a σ-algebra, where M is a finite set of models (including trends) and Θ is a finite set of parameters in the models. Let $\hat{\theta} = \hat{\theta}(X_1, X_2, \ldots, X_n)$ be an estimator of the true current estimate of liabilities θ and $X_1, X_2, \ldots, X_n$ be a random sample of size n from a probability distribution function with parameter θ, $f_\theta(\cdot)$ and $\theta \in \Theta$, the parameter space. The mean

[12] If you consider the transformation of the business to a "willing buyer" that is not putting the business in run-off, but considering it as a "going concern" then the risk margin could be calculated as $\mathrm{LRM} \approx \mathrm{CoC}\% \cdot \mathrm{SCR}_1$.

square – error (MSE) is the expectation of the squared – error loss in estimating θ by $\hat{\theta}$:

$$E\left[(\hat{\theta}-\theta)^2\right] = E\left[\left(E(\hat{\theta})-\theta\right)+\left(\hat{\theta}-E(\hat{\theta})\right)\right]^2$$
$$= (bias)^2 + V(\hat{\theta}). \tag{3a}$$

The last term in equation (3a) can be rewritten in terms of the sigma algebra F as

$$V(\hat{\theta}) = E_F\left(V(\hat{\theta} \mid F)\right) + V_F\left(E(\hat{\theta} \mid F)\right). \tag{3b}$$

Combining equations (3a) and (3b) we get

$$\text{MSE}(\hat{\theta}) = (bias)^2 + E_F\left(V(\hat{\theta} \mid F)\right) + V_F\left(E(\hat{\theta} \mid F)\right). \tag{3c}$$

In equation (3c) the first term, squared bias, is an issue for the supervisor, i.e. in the three Pillar approach outlined above, it is a Pillar II issue. The second term is the expected volatility, which is taken care of as a part of the SCR, but with a shorter time horizon. The third term represents the uncertainty in models, parameters and is the volatility of the level of the current estimate of liabilities. This term is the one that constitute the liability risk margin, LRM. Note that in non-life insurance this term is mainly a function of the liabilities, but in life insurance the liabilities could also be a function of the assets.
Assume that the time horizon $(0, T)$ is split in T uncorrelated time buckets, $1, 2, \ldots, T$, each representing a financial year (or accounting year), $[0, 1)$, $[1, 2), \ldots, [T-1, T)$. The uncertainty term in equation (3c) could be split up in two parts, one representing the time bucket $[0, 1)$ and the second the time bucket $[1, T)$. The first part will be included as a part of the SCR and the second will constitute the base for the risk margin.

3.3 Modelling SCR (risk categories, risk mitigation, diversification etc)

The MVL and the capital requirement, in terms of SCR, have somewhat different role in a solvency regime (IAIS (2006b)). The LRM is a safeguard to the current estimate of liabilities, CEL. The SCR provide further safeguard of the policyholders by protecting both the MVL and the MVA and their interaction. The SCR should be calibrated such that it could withstand current year claims experience in excess of current estimate and that assets still exceed the MVL at the end of a defined time horizon, say one year, with a certain degree of confidence, say 99.5%, or, that the available capital can withstand a range of predefined shocks or stress scenarios over the defined time horizon, cf. IAIS (2006b). In a risk based approach there need not be so much restrictions on the capital covering the "available solvency margin". Within the Solvency II project there will probably be a three-tier system of eligible capital.

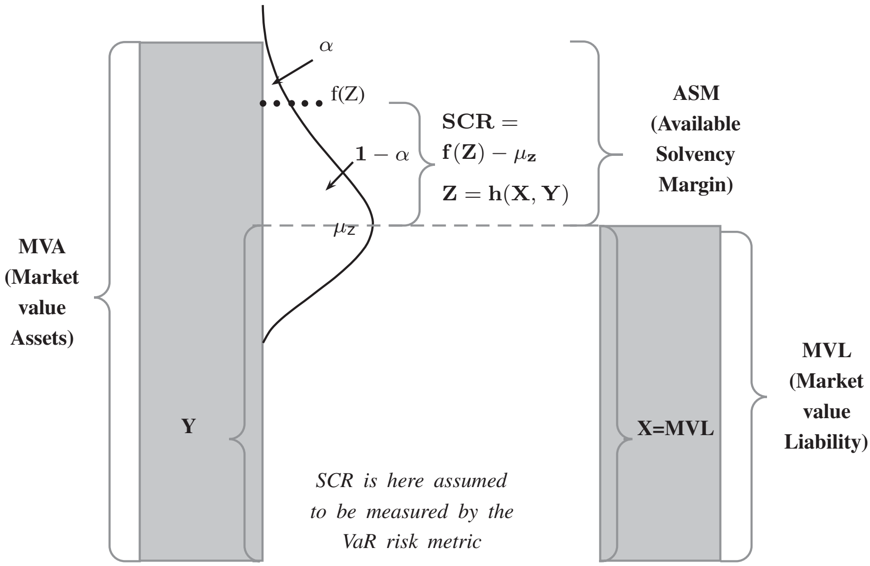

Figure 2. The SCR is calculated as the difference between a function of h(X,Y) and the mean of the distribution. The distribution is a function of the $X = \text{MVL}$ and the corresponding assets covering the liabilities (Y). *Source*: Sandström (2006b)

Let $X = \text{MVL}$ and Y be two stochastic variables, where Y is the assets covering the MVL, see Figure 2. The SCR is now defined as a function of $Z = h(X, Y)$. We can assume an unknown and probably skewed distribution function of Z with $\mu_Z = E[Z] = \text{MVL}$.

The solvency capital requirement is now defined as a stochastic variable $\text{SCR} = f(Z) - \mu_Z$, where $f(Z)$ is an appropriate risk measure, usually one takes the Value at Risk (VaR) or TailVaR, see below. The mean square error of an estimate of SCR could be evaluated in a similar way as in equation (3c) for a time horizon of $T = 1$ year. The function $h(X, Y)$ includes both the risks inherent in the liabilities (X) and the assets covering them (Y) as well as the interaction between the assets and liabilities.

To model the SCR, the IAA (2004) has proposed five main *risk categories*, mainly based on the Basel II Accord and insurance characteristics: (i) insurance risk (or underwriting risk) containing premium risks, claims reserving risk and catastrophe risks (CAT risks), (ii) credit risk, (iii) market risk, (iv) operational risk, and (v) liquidity risk. The insurance risk is associated both with the peril covered by the specific line of insurance business (fire, motor, liability, death, etc.) and with the specific processes associated with the conduct of the insurance business. Credit risk is the risk of default and change in the credit quality of issuers of securities, counterparties, and intermediaries, to which the company has an exposure (e.g. reinsurers). Market risks come from the level of volatility of market prices of assets. They involve the exposure to movements in the level of financial variables (e.g. stock prices, interest rates, exchange rates, etc.), but also the mismatch between assets and liabilities. Operational risks can be defined as the risks of loss resulting from inadequate or failed internal processes, people, and systems or from external events. Liquidity risk is the exposure to loss due to insufficient liquid assets being available. If a risk category is not possible to model and treated as a Pillar I risk, it should be treated as a Pillar II assessment, i.e. under the supervisory reviews process. These five risks are used both for solo entities but also for insurance groups or financial conglomerates. There is also a sixth main risk category that can be introduced for groups or conglomerates: group risk or participating risk. Examples of the latter type are an internal reinsurance program within an insurance group or the possibility that a bank is insuring its credit risk to an insurance company in the conglomerate.

The modeling is made in a top-down fashion. Each of the main risk categories is in the next step of modeling split up into sub risks, which in turn could be split up into sub-sub risks, etc. On the other hand, the calculation of the capital requirement is made in a bottom up approach, see Groupe Consultatif (2005), starting from the lowest level.

Diversification and mitigation are generic terms as they could be distinguished by its members. We start with the term “risk diversification” and ends up with the term “risk mitigation”. There is a clear connection between the two generic terms. The definition of the generic term diversification follows the proposal given by IAA in its answer and comments to IAIS’ paper IAIS (2006a), see IAA (2006).

Insurers are “***pooling***” risks in order to benefit from the “law of large numbers”. With pooling it is meant to aggregate similar risks that are similarly managed. “*The statistical concept is that mutually independent risks, when aggregated, will have experience that reflects a well behaved and measurable probability distribution function about the statistical mean. Note that aggregation of risks of significantly disparate size does not “(ensure) that volatility of future cash flows* *is at an economically sustainable level" if the largest risks accepted are too large relative to the size of the total financial resources of the insurer*." (IAA (2006))

"***Diversification*** *involves accepting risks that are not similar in order to benefit from the lessened correlation of contingent events*." (IAA (2006))

Hedging, or offsetting risks, involves accepting risks with a strong negative correlation as compared to diversification, that merely requires the absence of a strong positive correlation, (IAA (2006)) see above.

The most well-known ***risk mitigation*** technique in insurance context is reinsurance. But pooling, diversification and hedging also give risk mitigation benefits to the insurers.

3.4 Risk measures

In the pragmatic solution below we have used the notation of the standard deviation principle as a risk measure or risk metric in a base-line approach. Two popular risk metrics in the financial literature will be applied to this approach. They are the Value at Risk (VaR) and the TailVaR (expected shortfall), of which the latter is coherent for continuous random variables (see Acerbi-Tasche (2002)).

3.5 A pragmatic modelling approach

To model the capital requirements, as described by SCR in Figure 2, we have two main problems to deal with: the non-normality and the non-linearity.
Using a benchmark approach and tail correlations solves the non-linearity problems given in the pragmatic solution. Using Normal Power approximations can solve the non-normality problem.
We use the IAA-baseline as a start, IAA (2004). The general structure of the model is taken from a linear correlation structure. We use four risk categories and define the general baseline structure letting $C_i = k\sigma_i$, where k is a quantile function, which is clearly defined for a standard normal distribution. Then the total risk is

$$C = \Big(C_1^2 + C_2^2 + C_3^2 + C_4^2 + 2\rho_{12}C_1C_2 + 2\rho_{13}C_1C_3 + 2\rho_{14}C_1C_4 + 2\rho_{23}C_2C_3 + 2\rho_{24}C_2C_4 + 2\rho_{34}C_3C_4\Big)^{1/2}$$

	1	2	3	4
1	1	?	?	?
2		1	?	?
3			1	?
4				1

	1	2	3	4
1	1	?	?	1
2		1	?	1
3			1	1
4				1

linear correlation exact $\longrightarrow$

	1	2	3	4
1	1	1	1	1
2		1	1	1
3			1	1
4				1

Assume a situation where we have the left hand matrix with unknown correlations. Assume that we believe in that the fourth risk category is fully correlated with the other three, as described in the middle matrix. If the linear correlation were exact then we would have the right hand matrix, i.e. if the risks are fully correlated then $C = C_1 + C_2 + C_3 + C_4$. But this is not always what we believe in! In the Müller report (1997) the early version of the NAIC risk based system was described: *Once all RBC values of the individual categories have been calculated they are combined into the total RBC. For this the individual values are, however, not simply added up but compensation is made because not all risks will cause losses simultaneously. If it is assumed that both asset risk and interest rate risk ($C1$ and $C3$) are completely correlated and the technical risk ($C2$) is not related to either of them and in addition that the business risk ($C4$) is completely correlated with the other three risks this will result in a total RBC in life insurance* (RBC_{LV}) *as follows:*

$$\mathrm{RBC}_{LV} := C_4 + \sqrt{C_2^2 + (C_1 + C_3)^2}\,.$$

The "Benchmark approach" is a pragmatic solution for non-linear relationships. In terms of the matrices above and the additional assumption that first and third risks are fully correlated and that the second risk is uncorrelated with the first and third risks could be described as follows.

	1	2	3	4
1	1	?	?	?
2		1	?	?
3			1	?
4				1

	1	2	3	4
1	1	?	?	1
2		1	?	1
3			1	1
4				1

benchmark approach mode structure $\longrightarrow$

	1	2	3	4
1	1	0	1	1
2		1	0	1
3			1	1
4				1

We do as follows. The fourth risk is fully correlated with all the other three, i.e. $\rho_{4,(123)} = 1$ giving us the following pragmatic structure $C = C_4 + C_{(123)}$. Consider now $C_{(123)}$. The second risk is uncorrelated with the two other, i.e. $C_2^2 + C_{(13)}^2$. The last risk is $C_{(13)}^2 = (C_1 + C_3)^2$ since they are fully correlated. This gives us the RBC-structure above! Note that each main risk category can be thought of as consisting of different sub-risks; for example we can have the following structure for risk C_2: $C_2^2 = C_{21}^2 + (C_{22} + C_{23} + C_{24})^2 + 2\rho_{21,25} C_{21} C_{25} + C_{25}^2$.

3.5.1 Non-normality

Using a first order "normal power approximation" enables us to a solution to the non-normality problem. VaR (quantile) and TailVaR (Expected Shortfall)[13] of a skew standard distribution $F(.)$ can be expressed in terms of the VaR (quantile) and TailVaR of a standard normal distribution $\Phi(.)$. For the VaR case see Sandström (2005), Beard et al (1984), and Daykin et al (1994) and for the TailVaR case see Christoffersen & Goncalves (2005) and Giamouridis (2006), see also Sandström (2006a) for a sketch of proof.

The factor k depends on the skewness (γ) in the original distribution $F(.)$ and skewness is measured by $\gamma_i = \frac{E\left((Y_i - \mu_i)^3\right)}{\sigma_i^3}$, $i = 1, .., k$, and $\gamma_i \geq 0$. For different α and standard normal quantile $k_{1-\alpha}$ we get the following quantile for the VaR risk metric $k_{1,1-\alpha}(\gamma) = k_{1-\alpha} + \gamma(k_{1-\alpha}^2 - 1)/6$ and for the TailVaR risk metric $k_{2,1-\alpha}(\gamma) = \frac{1}{R(k_{1-\alpha})}\left(1 + \gamma\frac{k_{1-\alpha}^3}{6}\right)$, where $R(k_{1-\alpha})$ is Mills ratio. For $\alpha = 0.005$ we get $k_{1,0.995}(\gamma) = 2.58 + 0.94\gamma$ and $k_{2,0.995}(\gamma) = 2.89 + 8.30\gamma$, respectively.

To be more pragmatic: even if we have started with $C_* = k(\gamma_1)\sigma_*$ we may choose to let C_* be the result of scenarios or stress tests (i.e. the capital charge based on stress tests).

3.5.2 Non-normality and calibration

In the European Solvency II project the "target measure" is the Solvency Capital Requirement, SCR. This is a theoretical capital level that an insurance undertaking shall fulfil.

The European Commission (COM) has stated that *"The parameters in the SCR should be calibrated in such a way that the quantifiable risks to which an institution with a diversified portfolio of risks is exposed are taken into account and based on the amount of economic capital corresponding to a ruin probability of 0.5% (Value at Risk of 99.5%) and a one year time horizon. . . . The methods used to check that this level is effective must be defined. The SCR should be based*

[13] We assume that the distributions are continuous.

on a going-concern basis. These principles shall apply regardless of whether a standard formula or an internal model is used.”, COM (2006, para 17).
During the late spring 2006 the European Supervisory body, CEIOPS[14], together with the local supervisory authorities conducted a quantitative impact study (QIS). The aim with this QIS was to test a proposed model for calculating the SCR. In the study each risk charge in the model was intuitively and crude aimed at the 99.5 percentile, and then aggregated to the overall target, SCR. In the technical specifications to the QIS[15] (No. 2) it is stated a Calibration Approach in paragraphs 1.9–1.10:

(1.9) *The parameters used in the MCR and SCR reflect an initial, tentative calibration. Prior to collecting data from the exercise and other sources, CEIOPS cannot make assertions about the appropriateness of this calibration. The ‘target’ standard is TailVaR at an equivalent level of prudence to VaR 99.5%. A broad assumption has been made that TailVaR 99% would meet this objective, and this is reflected in certain SCR parameters.*

(1.10) *CEIOPS recognises that a coherent approach will be needed to ensure capital requirements are calibrated appropriately. For example, within the standard formula, each risk module will need to be calibrated to a consistent prudential standard. The aggregation process will then need to ensure that the overall SCR charge is calibrated to the same standard (e.g. with appropriate adjustments for cross-risk diversification effects). Such an approach to calibration would also facilitate the use of partial internal models for the SCR.*

To simplify the discussion, we only use two risk categories giving rise to two risk charges C_i, $i = 1, 2$ etc. This does not change the generality of the results. All risk charges are assumed to have a distribution that is either symmetric (standard normal) or positively skewed. If the risk charges are having distribution functions with $\gamma_i \geq 0$ then the skewness of the total risk charge distribution will be positively skewed: $\gamma_{\mathrm{SCR}} \geq 0$ with equality only if all distribution functions are normal distributed. The use of the normal power approximation is illustrated in Figure 3.

[14]CEIOPS: Committee of European Insurance and Occupational Pensions Supervisors, for more information see `www.ceiops.org`
[15]Documentation about QIS 2 can be found on CEIOPS website, see footnote 1.

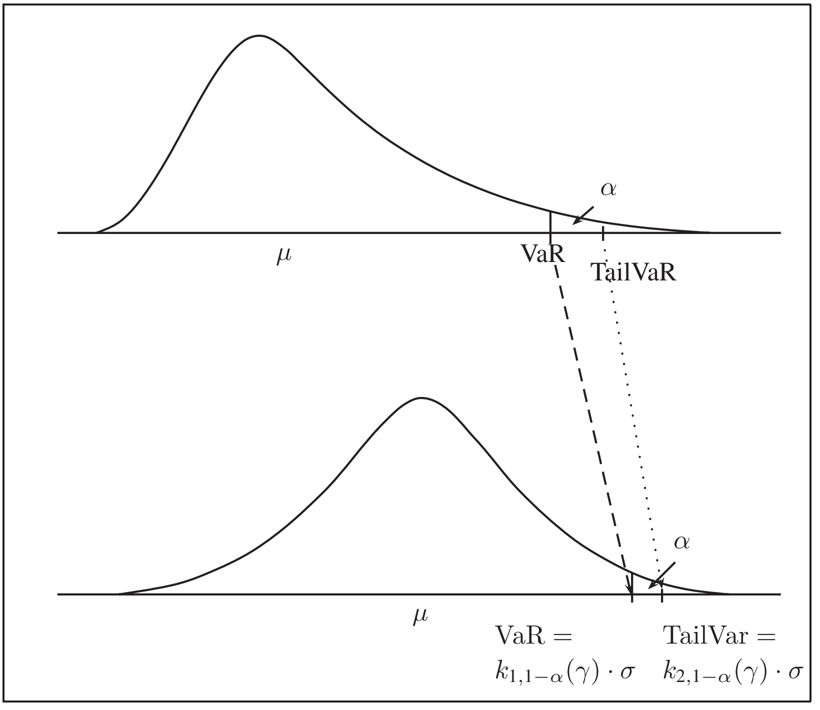

Figure 3. The quantile, VaR, and the tail expectation, TailVaR, redefined in terms of the standard normal distribution.

Assume

$$\mathrm{SCR}^2 = C_1^2 + C_2^2 + 2\rho_{12} C_1 C_2\,, \tag{4}$$

where the capital charges are seen as linear correlation functions, i.e. $C_i = k_{1-\alpha}\sigma_i$, $i = 1, 2$, where $k_{1-\alpha}$ is the percentile of the standard normal distribution. As we look at one-sided confidence intervals we have, as an example, $k_{0.995} = 2.58$. For illustrative purposes we assume that risk 1 is normal distributed (skewness: $\gamma_1 = 0$) and risk 2 is skewed distributed, e.g. Lognormal, (skewness: $\gamma_2 > 0$). This would imply that the SCR has a distribution that is less skewed than that for risk 2: $0 < \gamma_{\mathrm{SCR}} < \gamma_2$.

In the traditional NP approximation, the $1-\alpha$ percentile of a skew distribution is written in terms of the $1-\alpha$ percentile of the standard normal distribution with a correction for the skewness: $\mu + k_{1,1-\alpha}(\gamma) \cdot \sigma$, where $k_{1,1-\alpha}(\gamma)$ is a new percentile of the standard normal distribution and a function of the skewness in the original distribution. The NP approximation can be generalized so that the tail expectation

of a skew distribution also can be written in terms of the tail expectation of the standard normal distribution with a correction for the skewness: $\mu + k_{2,1-\alpha}(\gamma) \cdot \sigma$, where $k_{2,1-\alpha}(\gamma)$ is a new percentile of the standard normal distribution and a function of the skewness in the original distribution.
If each of the risk charges C_i, $i = 1, 2$, is calculated with $1 - \alpha$ confidence then the SCR has to be calibrated as

$$\mathrm{SCR}^2 = f_1^2 \cdot C_1^2 + f_2^2 \cdot C_2^2 + f_1 f_2 2\rho_{12} \cdot C_1 \cdot C_2 \tag{5}$$

to get the same confidence of $1 - \alpha$, i.e with a ruin probability of $\alpha\%$.
The factors f in equation (5) are calibration factors and they are all positive and usually around 1. Equation (5) holds both for VaR and TailVaR risk metrics and for a general number of risk charges; see also Sandström (2006a).
The general form of the calibration factors are

$$f_{i,\mathrm{VaR}} = \frac{k_{1,1-\alpha}(\gamma_{\mathrm{SCR}})^2}{k_{1,1-\alpha}(\gamma_i)^2} \tag{6a}$$

and

$$f_{i,\mathrm{TailVar}} = \frac{k_{2,1-\alpha}(\gamma_{SCR})^2}{k_{2,1-\alpha}(\gamma_i)^2} \tag{6b}$$

respectively for the VaR and TailVaR.
Comparing equation (5) with (4) shows that the calibration could be done by introducing calibration factors (> 0, usually > 1 or < 1). These calibration factors are ratios of functions of the skewness in the distributions involved in the calculation of the SCR, and also in the skewness of the distribution of SCR!
Note that the approach given by equation (5) can be used irrespective if the risk charges are calculated by distributional assumptions, as a result of a stress test or an internal model. The only assumption that must be fulfilled is that the calculations are made “intuitively and crude” at the $1 - \alpha$ percentile for one of the risk metrics.

3.5.3 Non-linearity

In assessing diversification effects IAA (2004) has proposed the use of Copulas as they can recognise dependencies that change in the tail of the distributions. Extreme events and tail dependencies are important for the insurance industry. In Groupe Consultatif (2005) it is proposed as a pragmatic solution to adjust

the correlation matrix with tail correlations instead of using the more complex Copulas approach.
The use of correlations as a dependence measure should be done very carefully. Copulas go beyond the linear dependence structures described by correlations and there are various pitfalls in the use of correlations and copulas (see e.g. Embrechts, McNeil & Straumann (2002)). Moreover, observe that VaR undergoes a phase transition from subadditivity to superadditivity when one changes from a finite mean model to an infinite mean model (see Embrechts, Neshlehova & Wüthrich (2006) and Alink, Löwe & Wüthrich (2004)). It is exactly this property, which has caused major concerns in the banking supervision for modelling operational risks.

4 Accounting

We have not discussed the important concept of accounting yet. Not to burden the insurance undertaking with both a statutory requirement and an accounting requirement it would be desirable to have one reporting system that could be used by both purposes. The International Accounting Standard Board (IASB[16]) has been working towards a consistent approach to insurance accounting. The main difference in this approach is its functional view (on insurance contracts) as compared to earlier approaches that have been on the institutional view (on insurance companies). The work done by IASB has been in coordination with the U.S. Financial Accounting Standard Board (FASB[17]). A presentation of IASB's work is given in Wright (2006).

[16]For more information see its website: `www.iasb.org`
[17]For more information see its website: `www.fasb.org`

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Abstract

The paper emphasizes that in constructing solvency systems a discussion of four fundamental issues is needed: the valuation of assets and liabilities, the risk margin for uncertainty in liabilities and assets, the risk measures and the modeling (risk categories, risk mitigation, diversification, etc.). It presents a historical review of solvency systems and some pragmatic solutions.

Zusammenfassung

Im Papier wird hervorgehoben, dass die Ausarbeitung eines Solvenzsystems auf vier fundamentalen Elementen beruht: Die Bewertung von Assets und Verpflichtungen, die Risikomarge für die Unsicherheit in den Assets und den Verpflichtungen, das Risikomass und die Modellierung (Risikokategorien, Risikominderung, Diversifikation, ...). Es werden ein historischer Überblick über Solvenzsysteme, sowie einige pragmatische Lösungen präsentiert.

Résumé

L'article souligne qu'à la base de l'élaboration de systèmes de solvabilté une discussion de quatre éléments fondamentaux est nécessaire. Il s'agit de l'évaluation des biens et des engagements, la marge de risque reflétant l'incertitude liée aux biens et aux engagements, la mesure du risque et finalement la modélisation (les catégories de risques, la mitigation des risques, la diversification, etc.). L'article présente un survol historique des systèmes de solvabilité ainsi que quelques solutions pragmatiques.